Manipulator Rigid Body Kinematic Equation & Equation of Motion Derivation

Kinematic Relations for Revolute and Prismatic Joints

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Homogeneous Transformations

Homogenous transformations permit the result of a sequence of rotational and translational operations applied to an object to be described by multiply the fundamental operations together.

Rotational Homogeneous Transformations

```
rotx[th_] := {{1, 0, 0, 0}, {0, Cos[th], -Sin[th], 0},
  {0, Sin[th], Cos[th], 0}, {0, 0, 0, 1}}
MatrixForm[rotx[β]]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \mathrm{Cos}[\beta] & -\mathrm{Sin}[\beta] & 0 \\ 0 & \mathrm{Sin}[\beta] & \mathrm{Cos}[\beta] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
roty[th_] := {{Cos[th], 0, Sin[th], 0},
  {0, 1, 0, 0}, {-Sin[th], 0, Cos[th], 0}, {0, 0, 0, 1}}
MatrixForm[roty[β]]
```

General::spell1 : Possible spelling error: new symbol name "roty" is similar to existing symbol "rotx".

$$\begin{pmatrix} \mathrm{Cos}[\beta] & 0 & \mathrm{Sin}[\beta] & 0 \\ 0 & 1 & 0 & 0 \\ -\mathrm{Sin}[\beta] & 0 & \mathrm{Cos}[\beta] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
rotz[th_] := {{Cos[th], -Sin[th], 0, 0},
  {Sin[th], Cos[th], 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
MatrixForm[rotz[β]]
```

General::spell : Possible spelling error: new symbol name "rotz" is similar to existing symbols {rotx, roty}.

$$\begin{pmatrix} \text{Cos}[\beta] & -\text{Sin}[\beta] & 0 & 0 \\ \text{Sin}[\beta] & \text{Cos}[\beta] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

■ Translational Homogeneous Transformations

```
transx[th_] := {{1, 0, 0, th}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}
MatrixForm[transx[a]]
```

$$\begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
transy[th_] := {{1, 0, 0, 0}, {0, 1, 0, th}, {0, 0, 1, 0}, {0, 0, 0, 1}}
MatrixForm[transy[b]]
```

General::spell1 : Possible spelling error: new symbol name "transy" is similar to existing symbol "transx".

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
transz[th_] := {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, th}, {0, 0, 0, 1}}
MatrixForm[transz[c]]
```

General::spell : Possible spelling error: new symbol name "transz" is similar to existing symbols {transx, transy}.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
trans[thx_, thy_, thz_] :=
 {{1, 0, 0, thx}, {0, 1, 0, thy}, {0, 0, 1, thz}, {0, 0, 0, 1}}
MatrixForm[trans[a, b, c]]
```

General::spell : Possible spelling error: new symbol name "trans" is similar to existing symbols {transx, transy, transz}.

$$\begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- **Examples**

```
MatrixForm[transx[a] . transy[b] . transz[c]]
```

$$\begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
MatrixForm[transy[b] . transz[c] . transx[a]]
```

$$\begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
MatrixForm[rotx[a] . roty[b] . rotz[c]]
```

$$\begin{pmatrix} \text{Cos}[b]\,\text{Cos}[c] & -\text{Cos}[b]\,\text{Sin}[c] & \cdots \\ \text{Cos}[c]\,\text{Sin}[a]\,\text{Sin}[b] + \text{Cos}[a]\,\text{Sin}[c] & \text{Cos}[a]\,\text{Cos}[c] - \text{Sin}[a]\,\text{Sin}[b]\,\text{Sin}[c] & -\text{Cos} \\ -\text{Cos}[a]\,\text{Cos}[c]\,\text{Sin}[b] + \text{Sin}[a]\,\text{Sin}[c] & \text{Cos}[c]\,\text{Sin}[a] + \text{Cos}[a]\,\text{Sin}[b]\,\text{Sin}[c] & \text{Cos} \\ 0 & 0 & \end{pmatrix}$$

```
MatrixForm[roty[b] . rotz[c] . rotx[a]]
```

$$\begin{pmatrix} \text{Cos}[b]\,\text{Cos}[c] & \text{Sin}[a]\,\text{Sin}[b] - \text{Cos}[a]\,\text{Cos}[b]\,\text{Sin}[c] & \text{Cos}[a]\,\text{Sin}[b] + \text{Cos}[b]\,\text{Sin} \\ \text{Sin}[c] & \text{Cos}[a]\,\text{Cos}[c] & -\text{Cos}[c]\,\text{Sin}[a \\ -\text{Cos}[c]\,\text{Sin}[b] & \text{Cos}[b]\,\text{Sin}[a] + \text{Cos}[a]\,\text{Sin}[b]\,\text{Sin}[c] & \text{Cos}[a]\,\text{Cos}[b] - \text{Sin}[a]\,\text{Sin} \\ 0 & 0 & 0 \end{pmatrix}$$

```
MatrixForm[Inverse[transx[a]]]
```

$$\begin{pmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
MatrixForm[Inverse[transx[a] . transy[b] . transz[c]]]
```

$$\begin{pmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{MatrixForm}\left[\text{trans}[4, -3, 7] \,.\, \text{roty}\left[\frac{\pi}{2}\right] \,.\, \text{rotz}\left[\frac{\pi}{2}\right]\right]$$

$$\begin{pmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
MatrixForm[trans[0, 0, z] . rotz[β]]
MatrixForm[rotz[β] . trans[0, 0, z]]
```

$$\begin{pmatrix} \text{Cos}[\beta] & -\text{Sin}[\beta] & 0 & 0 \\ \text{Sin}[\beta] & \text{Cos}[\beta] & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\beta] & -\text{Sin}[\beta] & 0 & 0 \\ \text{Sin}[\beta] & \text{Cos}[\beta] & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
MatrixForm[trans[x, 0, 0] . rotz[β]]
MatrixForm[rotz[β] . trans[x, 0, 0]]
```

$$\begin{pmatrix} \text{Cos}[\beta] & -\text{Sin}[\beta] & 0 & x \\ \text{Sin}[\beta] & \text{Cos}[\beta] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\beta] & -\text{Sin}[\beta] & 0 & x\,\text{Cos}[\beta] \\ \text{Sin}[\beta] & \text{Cos}[\beta] & 0 & x\,\text{Sin}[\beta] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Specification Matrices (A-matrix) for Joints

Specification matrices permits the description of one coordinate frame (orientation and position) relative to another. This is useful for the tree topology robot which the manipulator represents. Derivation of this matrix is performed by following a specific sequence of operations as described in Reference 1, pages 50-55.

- **Revolute Joints**
- **Prismatic Joints**

Manipulator Kinematics

Geometric layout of manipulator is displayed below:

Link Parameters

Specification Matrices

Follows

```
a1 = aa[θ1, d1, aa1, α1];
a2 = aa[θ2, d2, aa2, α2];
a3 = aa[θ3, d3, aa3, α3];
a4 = aa[θ4, d4, aa4, α4];
a5 = aa[θ5, d5, aa5, α5];
a6 = aa[θ6, d6, aa6, α6];
MatrixForm[a1]
MatrixForm[a2]
MatrixForm[a3]
MatrixForm[a4]
MatrixForm[a5]
MatrixForm[a6]
```

General::spell1 :
Possible spelling error: new symbol name "θ1" is similar to existing symbol "α1".

General::spell1 :
Possible spelling error: new symbol name "θ2" is similar to existing symbol "α2".

General::spell1 :
Possible spelling error: new symbol name "θ3" is similar to existing symbol "α3".

General::stop :
Further output of General::spell1 will be suppressed during this calculation.

$$\begin{pmatrix} \text{Cos}[\theta 1] & 0 & \text{Sin}[\theta 1] & 0 \\ \text{Sin}[\theta 1] & 0 & -\text{Cos}[\theta 1] & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 2] & -\text{Sin}[\theta 2] & 0 & \frac{313\,\text{Cos}[\theta 2]}{5} \\ \text{Sin}[\theta 2] & \text{Cos}[\theta 2] & 0 & \frac{313\,\text{Sin}[\theta 2]}{5} \\ 0 & 0 & 1 & \frac{69}{10} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 3] & 0 & \text{Sin}[\theta 3] & 0 \\ \text{Sin}[\theta 3] & 0 & -\text{Cos}[\theta 3] & 0 \\ 0 & 1 & 0 & \frac{177}{10} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 4] & 0 & -\text{Sin}[\theta 4] & 46\,\text{Cos}[\theta 4] \\ \text{Sin}[\theta 4] & 0 & \text{Cos}[\theta 4] & 46\,\text{Sin}[\theta 4] \\ 0 & -1 & 0 & \frac{9}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 5] & 0 & \text{Sin}[\theta 5] & 0 \\ \text{Sin}[\theta 5] & 0 & -\text{Cos}[\theta 5] & 0 \\ 0 & 1 & 0 & \frac{397}{10} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 6] & -\text{Sin}[\theta 6] & 0 & \frac{31\,\text{Cos}[\theta 6]}{10} \\ \text{Sin}[\theta 6] & \text{Cos}[\theta 6] & 0 & \frac{31\,\text{Sin}[\theta 6]}{10} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

■ T6 Specification Matrix

The matrix with relates the end of the manipulator (concentrator) with respect to the base coordinate frame is given as

```
t6 = a1 . a2 . a3 . a4 . a5 . a6;
MatrixForm[t6]
MatrixForm[t6 /. {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3,
   Cos[θ4] -> c4, Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
```

```
( Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Sin[θ2] Sin[θ3])
  Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] - Sin[θ1] Sin[θ2] Sin[θ3])
                                                    Cos[θ6] (Cos[θ4] Cos[θ5] (Cos[θ3
```

```
( c6 (c5 (c4 (c1 c2 c3 - c1 s2 s3) + s1 s4) + (-c1 c3 s2 - c1 c2 s3) s5) + (c4 s1 - (c1 c2
  c6 (c5 (c4 (c2 c3 s1 - s1 s2 s3) - c1 s4) + (-c3 s1 s2 - c2 s1 s3) s5) + (-c1 c4 - (c2 c3
                    c6 (c4 c5 (c3 s2 + c2 s3) + (c2 c3 - s2 s3) s5) - (c3 s2 + c2 s3) s4
                                                   0
```

Note that columns 1 thru 3 of t6 resolves the end of the manipulator frame into the base frame. Column 4 specifies the origin location of the end of the manipulator frame in base coordinates. Also, note the t6 transforms in vector specified in the end of the manipulator (concentrator frame) coordinates into the base frame coordinates.

For the nominal configuration

```
MatrixForm[
 t6 /. {θ1 -> θ1n, θ2 -> θ2n, θ3 -> θ3n, θ4 -> θ4n, θ5 -> θ5n, θ6 -> θ6n}]
MatrixForm[
 N[t6 /. {θ1 -> θ1n, θ2 -> θ2n, θ3 -> θ3n, θ4 -> θ4n, θ5 -> θ5n, θ6 -> θ6n}]]
```

$$\begin{pmatrix} 0 & 1 & 0 & -\frac{107}{5} \\ 1 & 0 & 0 & \frac{527}{5} \\ 0 & 0 & -1 & -\frac{9}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1. & 0 & -21.4 \\ 1. & 0 & 0 & 105.4 \\ 0 & 0 & -1. & -4.5 \\ 0 & 0 & 0 & 1. \end{pmatrix}$$

Useing pre- and post- matrices for base and concentrator alignments. This yields (nearly) proper orientation alignment and position for concentrator.

```
(* Clear[d1,d2,d3,d4,d5,d6,aa1,aa2,aa3,aa4,aa5,aa6,t6] *)

aperXFMframe0 =
 {{-1, 0, 0, -21.4}, {0, -1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
MatrixForm[aperXFMframe0]

frame6XFMhotspot =
 {{0, -1, 0, 0}, {0, 0, 1, 0}, {-1, 0, 0, 0}, {0, 0, 0, 1}};
MatrixForm[frame6XFMhotspot]

MatrixForm[N[aperXFMframe0 . t6 . frame6XFMhotspot /.
   {θ1 -> θ1n, θ2 -> θ2n, θ3 -> θ3n, θ4 -> θ4n, θ5 -> θ5n, θ6 -> θ6n}]]
```

$$\begin{pmatrix} -1 & 0 & 0 & -21.4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & -1. & 0. \\ 0 & 1. & 0 & -105.4 \\ 1. & 0 & 0 & -4.5 \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

Now roll about gimbal5 -90 deg. Positional coordinates at hotspot (HS) should remain the same. This is close to the nominal configuration of the manipulator.

```
(* Clear[d1,d2,d3,d4,d5,d6,aa1,aa2,aa3,aa4,aa5,aa6,t6] *)
 MatrixForm[N[aperXFMframe0 . t6 . frame6XFMhotspot /. {θ1 -> θ1n, θ2 -> θ2n,
    θ3 -> θ3n, θ4 -> θ4n, θ5 -> (θ5n - 90 * Pi / 180), θ6 -> θ6n}]]
```

$$\begin{pmatrix} 1. & 0 & 0 & 0. \\ 0 & 1. & 0 & -105.4 \\ 0 & 0 & 1. & -4.5 \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

Now maneuver out the 4.5". This is the rough nominal configuration of the manipulator. But roll axis is off alignment.

```
(* Clear[d1,d2,d3,d4,d5,d6,aa1,aa2,aa3,aa4,aa5,aa6,t6] *)
MatrixForm[N[aperXFMframe0 . t6 . frame6XFMhotspot /.
   {θ1 -> θ1n, θ2 -> (4.123 * Pi / 180 + θ2n), θ3 -> (-4.123 * Pi / 180 + θ3n),
    θ4 -> θ4n, θ5 -> (θ5n - 1 * 90 * Pi / 180), θ6 -> (0 * Pi / 180 + θ6n)}]]
```

$$\begin{pmatrix} 1. & 0 & 0 & 0. \\ 0 & 1. & 0. & -105.238 \\ 0 & 0. & 1. & 0.000804593 \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

Now to add concentrator normal vector at concentrator/manipulator attach point to determine reflected sun location. The unit normal (-gradient) vector at the attach point in hot spot space is shown below as well as its transformation to base coordinates for the operational configuration. The unit sun vector defined in base & hot spot coordinates is also displayed. The reflected sun direction in hot spot coordinates is also determined below. Also determine reflected sun location on RAC image plane. This will be done using homogeneous coordinates for planes and points.

```
ClearAll[x, y]
vertexXFMbase =
 {{Sin[θ], Cos[θ], 0, f}, {Cos[θ], -Sin[θ], 0, 0}, {0, 0, -1, 0},
{0, 0, 0, 1}};
MatrixForm[vertexXFMbase]
```

$$\begin{pmatrix} \mathrm{Sin}[\theta] & \mathrm{Cos}[\theta] & 0 & f \\ \mathrm{Cos}[\theta] & -\mathrm{Sin}[\theta] & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
(* in this file base and aperture frames are identical *)
 (* was hsbase={{0},{d},{0},{1}}; *)
hsbase =
 {{d Sin[θpt] Sin[αpt]}, {-d Cos[θpt]}, {d Sin[θpt] Cos[αpt]}, {1}};
MatrixForm[hsbase]
```

*General::spell1 :
Possible spelling error: new symbol name "αpt" is similar to existing symbol "θpt".*

$$\begin{pmatrix} d\,\mathrm{Sin}[\alpha pt]\,\mathrm{Sin}[\theta pt] \\ -d\,\mathrm{Cos}[\theta pt] \\ d\,\mathrm{Cos}[\alpha pt]\,\mathrm{Sin}[\theta pt] \\ 1 \end{pmatrix}$$

```
hsvertex = vertexXFMbase . hsbase;
MatrixForm[hsvertex]
```

$$\begin{pmatrix} f - d\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\theta pt] + d\,\mathrm{Sin}[\alpha pt]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta pt] \\ d\,\mathrm{Cos}[\theta pt]\,\mathrm{Sin}[\theta] + d\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\alpha pt]\,\mathrm{Sin}[\theta pt] \\ -d\,\mathrm{Cos}[\alpha pt]\,\mathrm{Sin}[\theta pt] \\ 1 \end{pmatrix}$$

```
xvertex = hsvertex[[1, 1]]
yvertex = hsvertex[[2, 1]]
zvertex = hsvertex[[3, 1]]
```

f - d Cos[θ] Cos[θpt] + d Sin[αpt] Sin[θ] Sin[θpt]

*General::spell1 : Possible spelling error: new symbol name "yvertex" is
similar to existing symbol "xvertex".*

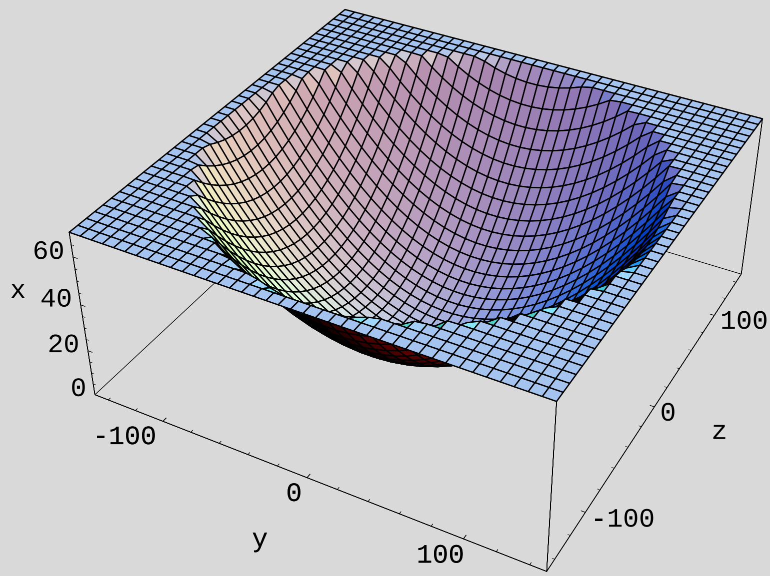
d Cos[θpt] Sin[θ] + d Cos[θ] Sin[αpt] Sin[θpt]

*General::spell : Possible spelling error: new symbol name "zvertex" is
similar to existing symbols {xvertex, yvertex}.*

-d Cos[αpt] Sin[θpt]

```
concentratorParabola[ypara_, zpara_] := (ypara^2 + zpara^2) / (4 * f)
Plot3D[
 concentratorParabola[y, z] /. f -> 70, {z, -150, 150}, {y, -150, 150},
 PlotRange -> {-0, 70}, PlotPoints -> 40, AxesLabel → {"y", "z", "x"}]
```

General::spell1 : Possible spelling error: new symbol name "zpara" is similar to existing symbol "ypara".



- SurfaceGraphics -

```
ans = Solve[
  concentratorParabola[hsvertex[[2, 1]], hsvertex[[3, 1]]] == xvertex, d]
```

$$\left\{\left\{d \to \left(2\left(-f\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\theta pt] + f\,\mathrm{Sin}[\alpha pt]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta pt] - f\sqrt{(\mathrm{Cos}[\theta]^2\,\mathrm{Cos}[\theta pt]^2 + \mathrm{Cos}[\theta pt]^2\,\mathrm{Sin}[\theta]^2 + \mathrm{Cos}[\alpha pt]^2\,\mathrm{Sin}[\theta pt]^2 + \mathrm{Cos}[\theta]^2\,\mathrm{Sin}[\alpha pt]^2\,\mathrm{Sin}[\theta pt]^2 + \mathrm{Sin}[\alpha pt]^2\,\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\theta pt]^2)}\right)\right) \Big/ (\mathrm{Cos}[\theta pt]^2\,\mathrm{Sin}[\theta]^2 + 2\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\theta pt]\,\mathrm{Sin}[\alpha pt]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta pt] + \mathrm{Cos}[\alpha pt]^2\,\mathrm{Sin}[\theta pt]^2 + \mathrm{Cos}[\theta]^2\,\mathrm{Sin}[\alpha pt]^2\,\mathrm{Sin}[\theta pt]^2)\right\},$$

$$\left\{d \to \left(2\left(-f\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\theta pt] + f\,\mathrm{Sin}[\alpha pt]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta pt] + f\sqrt{(\mathrm{Cos}[\theta]^2\,\mathrm{Cos}[\theta pt]^2 + \mathrm{Cos}[\theta pt]^2\,\mathrm{Sin}[\theta]^2 + \mathrm{Cos}[\alpha pt]^2\,\mathrm{Sin}[\theta pt]^2 + \mathrm{Cos}[\theta]^2\,\mathrm{Sin}[\alpha pt]^2\,\mathrm{Sin}[\theta pt]^2 + \mathrm{Sin}[\alpha pt]^2\,\mathrm{Sin}[\theta]^2\,\mathrm{Sin}[\theta pt]^2)}\right)\right) \Big/ (\mathrm{Cos}[\theta pt]^2\,\mathrm{Sin}[\theta]^2 + 2\,\mathrm{Cos}[\theta]\,\mathrm{Cos}[\theta pt]\,\mathrm{Sin}[\alpha pt]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta pt] + \mathrm{Cos}[\alpha pt]^2\,\mathrm{Sin}[\theta pt]^2 + \mathrm{Cos}[\theta]^2\,\mathrm{Sin}[\alpha pt]^2\,\mathrm{Sin}[\theta pt]^2)\right\}\right\}$$

```
dmag = N[ans /. {θ -> 70 Pi / 180, f -> 70, θ -> 70 Pi / 180, θpt -> 0 Pi / 180,
    αpt -> 0 Pi / 180}]

{{d → -212.772}, {d → 104.32}}
```

```
zvalvertex = N[hsvertex[[3, 1]] /.
   {θ -> 70 Pi / 180, dmag[[2]][[1]], θpt -> 0 Pi / 180, αpt -> 0 Pi / 180}]

0
```

```
yvalvertex = N[hsvertex[[2, 1]] /.
   {θ -> 70 Pi / 180, dmag[[2]][[1]], θpt -> 0 Pi / 180, αpt -> 0 Pi / 180}]

General::spell1 : Possible spelling error: new symbol name "yvalvertex" is
   similar to existing symbol "zvalvertex".

98.0291
```

```
xvalvertex = N[hsvertex[[1, 1]] /. {θ -> 70 Pi / 180, f -> 70,
    dmag[[2]][[1]], θpt -> 0 Pi / 180, αpt -> 0 Pi / 180}]

General::spell : Possible spelling error: new symbol name "xvalvertex" is
   similar to existing symbols {yvalvertex, zvalvertex}.

34.3203
```

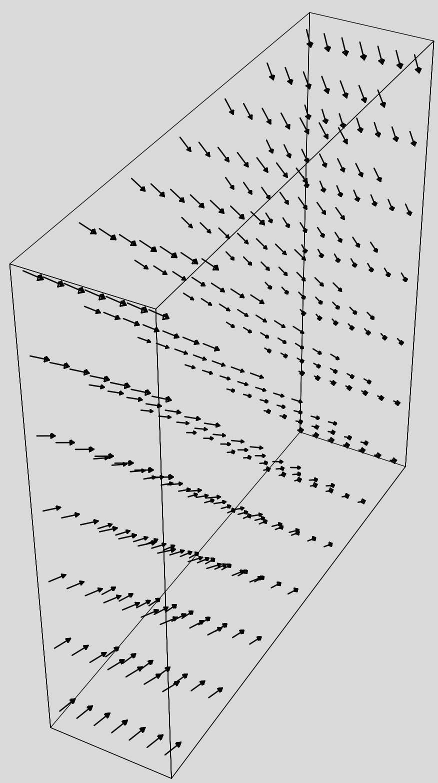
```
concentratorParabola[yvalvertex, 0] /. f -> 70

34.3203
```

```
concentratorParabolaLevelSurface[ypara_, zpara_, xpara_] :=
 (ypara^2 + zpara^2) / (4 * f) - xpara
concentratorParabolaLevelSurface[yy, zz, xx]
PlotGradientField3D[
 -concentratorParabolaLevelSurface[yy, zz, xx] /. f -> 70,
 {xx, 0, 70}, {yy, -150, 150}, {zz, -150, 150}, VectorHeads -> True]
(*z (ii,jj)=(x (ii,jj)^2+y (ii,jj)^2)/(4*focus);*)
```

General::spell : Possible spelling error: new symbol name "xpara" is similar to existing symbols {ypara, zpara}.

$$-xx + \frac{yy^2 + zz^2}{4\,f}$$



- Graphics3D -

```
concentratorParabolaGradient[yy_, zz_, xx_] := Grad[
  -concentratorParabolaLevelSurface[yy, zz, xx], Cartesian[xx, yy, zz]]
concentratorParabolaGradient[yy, zz, xx] // MatrixForm
vecvertex = concentratorParabolaGradient[yy, zz, xx] /.
  {xx -> 10000000, yy -> yvalvertex, zz -> 0, f -> 70}
veccnormvertex = N[Normalize[vecvertex]]
ArcTan[veccnormvertex[[1]], veccnormvertex[[2]]] * 180 / Pi
veccnormbase = Inverse[vertexXFMbase] .
   Append[veccnormvertex, 0] /. {θ -> 70 Pi / 180, f -> 70};
veccnormbase // MatrixForm
```

$$\begin{pmatrix} 1 \\ -\frac{yy}{2 f} \\ -\frac{zz}{2 f} \end{pmatrix}$$

```
{1, -0.700208, 0}
```

```
{0.819152, -0.573576, 0}
```

```
-35.
```

$$\begin{pmatrix} 0.573576 \\ 0.819152 \\ 0 \\ 0. \end{pmatrix}$$

```
ŝb = {-Cos[20 * Pi / 180], -Sin[20 * Pi / 180], 0, 0};
MatrixForm[ŝb]

(* was this
nĥs={Sin[35*Pi/180],Cos[35*Pi/180],0,0};
MatrixForm[nĥs] *)

(* now define negative gradient in
  hotspot (concentrator) reference frame using nominal
  configuration. This vector is fixed wrt this reference frame *)
nĥs = Inverse[N[aperXFMframe0 . t6 . frame6XFMhotspot /.
     {θ1 -> θ1n, θ2 -> (4.123 * Pi / 180 + θ2n), θ3 -> (-4.123 * Pi / 180 + θ3n),
      θ4 -> θ4n, θ5 -> (θ5n - 1 * 90 * Pi / 180), θ6 -> (0 * Pi / 180 + θ6n)}]] .
  veccnormbase;
MatrixForm[nĥs]

taperXFMhotspot = aperXFMframe0 . t6 . frame6XFMhotspot /.
  {θ1 -> θ1n, θ2 -> (4.123 * Pi / 180 + θ2n), θ3 -> (-4.123 * Pi / 180 + θ3n),
   θ4 -> θ4n, θ5 -> (θ5n - 1 * (90 + 30) * Pi / 180), θ6 -> (0 * Pi / 180 + θ6n)};
taperXFMhotspot // MatrixForm
n̂b = taperXFMhotspot . nĥs;
MatrixForm[n̂b]

qsunb = Quaternion[0, ŝb[[1]], ŝb[[2]], ŝb[[3]]];
qnb = Quaternion[0, n̂b[[1]], n̂b[[2]], n̂b[[3]]];
reflectedsunb = qnb ** qsunb ** qnb;
reflectedsunb = reflectedsunb / Abs[reflectedsunb];
FromQuaternion[reflectedsunb]
```

$$\begin{pmatrix} -\mathrm{Cos}[\frac{\pi}{9}] \\ -\mathrm{Sin}[\frac{\pi}{9}] \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0.573576 \\ 0.819152 \\ 0. \\ 0. \end{pmatrix}$$

$$\begin{pmatrix} 0.866025 & 0 & 0.5 & 0. \\ 0. & 1. & 0. & -105.238 \\ -0.5 & 0. & 0.866025 & 0.000804593 \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

$$\begin{pmatrix} 0.496732 \\ 0.819152 \\ -0.286788 \\ 0. \end{pmatrix}$$

$$(5.55112 \times 10^{-17} - 0.197633\,I) + 0.881697\,J - 0.428428\,K$$

Following shows that 16-bit resolution may be required from resolvers.

```
taperXFMhotspot = aperXFMframe0 . t6 . frame6XFMhotspot /.
  {θ1 -> (-60 * 0 * Pi / 180 + θ1 n), θ2 -> (4.123 * 1.225 * Pi / 180 + θ2 n),
   θ3 -> (-4.123 * 1.225 * Pi / 180 + θ3 n), θ4 -> (-60 * 0 * Pi / 180 + θ4 n),
   θ5 -> (θ5 n - 1 * (90 + 0) * Pi / 180), θ6 -> (0 * Pi / 180 + θ6 n)};
taperXFMhotspot // MatrixForm

thotspotXFMaper = Inverse[taperXFMhotspot];
thotspotXFMaper // MatrixForm

sĥs = thotspotXFMaper . ŝb;
MatrixForm[sĥs]

qsunhs = Quaternion[0, sĥs[[1]], sĥs[[2]], sĥs[[3]]];
qnhs = Quaternion[0, nĥs[[1]], nĥs[[2]], nĥs[[3]]];

reflectedsunhs = qnhs ** qsunhs ** qnhs;
reflectedsunhs = reflectedsunhs / Abs[reflectedsunhs];
FromQuaternion[reflectedsunhs]

reflectedsunvechs =
 {reflectedsunhs[[2]], reflectedsunhs[[3]], reflectedsunhs[[4]], 0};
reflectedsunvechs // MatrixForm
taperXFMhotspot . reflectedsunvechs // MatrixForm

(* image plane normal direction and distance from origin,
 in normal direction, relative to aperture space*)
racImagePlaneNormalb = {0, 1, 0, 0} ;
racImagePlaneNormalb // MatrixForm

(*transform to hot spot space*)
racImagePlaneNormalhs = racImagePlaneNormalb . taperXFMhotspot;
racImagePlaneNormalhs // MatrixForm

sunPointOnImagePlaneScale =
 -racImagePlaneNormalhs[[4]] / racImagePlaneNormalhs . reflectedsunvechs
sunPointOnImagePlanehs =
 trans[sunPointOnImagePlaneScale * reflectedsunvechs[[1]],
   sunPointOnImagePlaneScale * reflectedsunvechs[[2]],
   sunPointOnImagePlaneScale * reflectedsunvechs[[3]]] . {0, 0, 0, 1}
sunPointOnImagePlaneb = taperXFMhotspot . sunPointOnImagePlanehs
```

$$\begin{pmatrix} 1. & 0 & 0 & 0. \\ 0 & 1. & 0. & -105.157 \\ 0 & 0. & 1. & 1.0111 \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 0. \\ 0. & 1. & 0. & 105.157 \\ 0. & 0. & 1. & -1.0111 \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

$$\begin{pmatrix} -0.939693 \\ -0.34202 \\ 0. \\ 0. \end{pmatrix}$$

General::spell1 :
Possible spelling error: new symbol name "qnhs" is similar to existing symbol "nhs".

$$(0. - 5.55112 \times 10^{-17}\, I) + 1.\, J + 0.\, K$$

$$\begin{pmatrix} -5.55112 \times 10^{-17} \\ 1. \\ 0. \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -5.55112 \times 10^{-17} \\ 1. \\ 0. \\ 0. \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0. \\ 1. \\ 0. \\ -105.157 \end{pmatrix}$$

```
105.157
```

```
{-5.83738×10^-15, 105.157, 0., 1.}
```

```
{-5.83738×10^-15, 0., 1.0111, 1.}
```

Now to plot out sun image at RAC due to 30 deg roll motion. Note that result doesn't yield a straight line.

```
Clear[temp]
Array[temp, {1, 21}];

ŝb = {-Cos[20*Pi/180], -Sin[20*Pi/180], 0, 0};
MatrixForm[ŝb]

taperXFMhotspotnom = N[aperXFMframe0 . t6 . frame6XFMhotspot /.
   {θ1 -> θ1n, θ2 -> (4.123*Pi/180 + θ2n), θ3 -> (-4.123*Pi/180 + θ3n),
    θ4 -> θ4n, θ5 -> (θ5n - 1*90*Pi/180), θ6 -> (0*Pi/180 + θ6n)}];
thotspotXFMapernom = Inverse[taperXFMhotspotnom];
baseXFMvertex = Inverse[vertexXFMbase] /. {θ -> 70 Pi/180, f -> 70};

dmag = N[ans /. {θ -> 70 Pi/180,
    f -> 70, θ -> 70 Pi/180, θpt -> +0 Pi/180, αpt -> -0 Pi/180}]
ptnvertex = N[hsvertex /. {θ -> 70 Pi/180,
    f -> 70, dmag[[2]][[1]], θpt -> +0 Pi/180, αpt -> -0 Pi/180}];
ptnvertex // MatrixForm
zvalvertex = ptnvertex[[3, 1]]
yvalvertex = ptnvertex[[2, 1]]
xvalvertex = ptnvertex[[1, 1]]
(* concentratorParabola[yvalvertex,zvalvertex]/.f->70
Inverse[vertexXFMbase].numhsvertex/.
   {θ->70 Pi/180,f->70}//MatrixForm *)

(* yvalvertex=10*yvalvertex;zvalvertex=10;
xvalvertex=concentratorParabola[yvalvertex,zvalvertex]/.f->70

ptnvertex={xvalvertex,yvalvertex,zvalvertex,1} *)
ptnhs = thotspotXFMapernom . baseXFMvertex . ptnvertex

concGradientvertex = concentratorParabolaGradient[yy, zz, xx] /.
  {xx -> xvalvertex, yy -> yvalvertex, zz -> zvalvertex, f -> 70}
concGradientvertex = N[Normalize[concGradientvertex]]
ArcTan[concGradientvertex[[1]], concGradientvertex[[2]]]*180/Pi
concGradientbase = thotspotXFMapernom . baseXFMvertex .
```

```
  Append[concGradientvertex, 0] /. {θ -> 70 Pi / 180, f -> 70};

nĥs = concGradientbase;
MatrixForm[nĥs]
qptnhs = Quaternion[1, ptnhs[[1]], ptnhs[[2]], ptnhs[[3]]];
qnhs = Quaternion[0, nĥs[[1]], nĥs[[2]], nĥs[[3]]];

For[nn = 1, nn < 22, nn++,
taperXFMhotspot = aperXFMframe0 . t6 . frame6XFMhotspot /.
   {θ1 -> (-60 * 0 * Pi / 180 + θ1n), θ2 -> (4.123 * 1 * Pi / 180 + θ2n),
    θ3 -> (-4.123 * 1 * Pi / 180 + θ3n), θ4 -> (-60 * 0 * Pi / 180 + θ4n),
    θ5 -> (θ5n - 1 * (90 + 3 * nn - 33) * Pi / 180), θ6 -> (0 * Pi / 180 + θ6n)};
thotspotXFMaper = Inverse[taperXFMhotspot];
sĥs = thotspotXFMaper . sb̂;

qsunhs = Quaternion[0, sĥs[[1]], sĥs[[2]], sĥs[[3]]];

reflectedsunhs = qnhs ** qsunhs ** qnhs;
reflectedsunhs = reflectedsunhs / Abs[reflectedsunhs];
FromQuaternion[reflectedsunhs];

reflectedsunvechs =
  {reflectedsunhs[[2]], reflectedsunhs[[3]], reflectedsunhs[[4]], 0};
taperXFMhotspot . reflectedsunvechs;

(* image plane normal direction and distance from origin,
  in normal direction, relative to aperture space*)
racImagePlaneNormalb = {0, 1, 0, 0} ;

(*transform to hot spot space*)
racImagePlaneNormalhs = racImagePlaneNormalb . taperXFMhotspot;

sunPointOnImagePlaneScale = (-racImagePlaneNormalhs[[4]] -
     racImagePlaneNormalhs[[{1, 2, 3}]] . ptnhs[[{1, 2, 3}]]) /
   racImagePlaneNormalhs[[{1, 2, 3}]] . reflectedsunvechs[[{1, 2, 3}]];
sunPointOnImagePlaneScale = sunPointOnImagePlaneScale[[1]];
sunPointOnImagePlanehs =
  trans[sunPointOnImagePlaneScale * reflectedsunvechs[[1]],
    sunPointOnImagePlaneScale * reflectedsunvechs[[2]],
    sunPointOnImagePlaneScale * reflectedsunvechs[[3]]] . ptnhs;
sunPointOnImagePlaneb = taperXFMhotspot . sunPointOnImagePlanehs;

temp[1, nn] = Transpose[sunPointOnImagePlaneb][[1]];
]
ListPlot[{{temp[1, 1][[1]], temp[1, 1][[3]]},
```

```
   {temp[1, 2][[1]], temp[1, 2][[3]]}, {temp[1, 3][[1]], temp[1, 3][[3]]},
   {temp[1, 4][[1]], temp[1, 4][[3]]}, {temp[1, 5][[1]], temp[1, 5][[3]]},
   {temp[1, 6][[1]], temp[1, 6][[3]]}, {temp[1, 7][[1]], temp[1, 7][[3]]},
   {temp[1, 8][[1]], temp[1, 8][[3]]}, {temp[1, 9][[1]], temp[1, 9][[3]]},
   {temp[1, 10][[1]], temp[1, 10][[3]]},
   {temp[1, 11][[1]], temp[1, 11][[3]]},
   {temp[1, 12][[1]], temp[1, 12][[3]]},
   {temp[1, 13][[1]], temp[1, 13][[3]]},
   {temp[1, 14][[1]], temp[1, 14][[3]]},
   {temp[1, 15][[1]], temp[1, 15][[3]]},
   {temp[1, 16][[1]], temp[1, 16][[3]]},
   {temp[1, 17][[1]], temp[1, 17][[3]]},
   {temp[1, 18][[1]], temp[1, 18][[3]]},
   {temp[1, 19][[1]], temp[1, 19][[3]]},
   {temp[1, 20][[1]], temp[1, 20][[3]]},
   {temp[1, 21][[1]], temp[1, 21][[3]]}},
 PlotJoined -> True, AspectRatio -> Automatic]
```

$$\begin{pmatrix} -\mathrm{Cos}[\frac{\pi}{9}] \\ -\mathrm{Sin}[\frac{\pi}{9}] \\ 0 \\ 0 \end{pmatrix}$$

{{d → −212.772}, {d → 104.32}}

$$\begin{pmatrix} 34.3203 \\ 98.0291 \\ 0 \\ 1. \end{pmatrix}$$

0

98.0291

34.3203

{{0.}, {0.91765}, {−0.000804593}, {1.}}

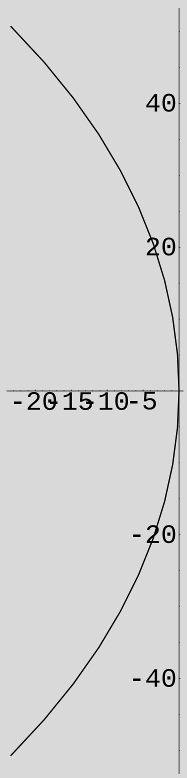
{1, −0.700208, 0}

```
{0.819152, -0.573576, 0}
```

```
-35.
```

$$\begin{pmatrix} 0.573576 \\ 0.819152 \\ 0. \\ 0. \end{pmatrix}$$

General::spell1 : Possible spelling error: new symbol name "qptnhs" is similar to existing symbol "ptnhs".




```
- Graphics -
```

```
temp[1, 1]
```

```
{-23.3831, 0., 50.6907, 1.}
```

Inverse Kinematics

Desired Joint Coordinates from Desired Specification (T_6) Matrix

Here the derivation for the required joint angles given the required concentrator position and orientation will be performed. The concentrator specification matrix is assumed to be given in the following form:

```
t6reqd =
 {{nx, ox, ax, px}, {ny, oy, ay, py}, {nz, oz, az, pz}, {0, 0, 0, 1}};
MatrixForm[t6reqd]
```

$$\begin{pmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The joint coordinates are now determined. First, the following equation is computed:

```
Clear[d1, d2, d3, d4, d5, d6, aa1, aa2, aa3, aa4, aa5, aa6, t6]
α1 = 90 * π/180; aa1 = 0; d1 = 0;
α2 = 0 * π/180;
α3 = 90 * π/180; aa3 = 0;
α4 = -90 * π/180;
α5 = 90 * π/180; aa5 = 0;
α6 = 0 * π/180; d6 = 0;

a1 = aa[θ1, d1, aa1, α1];
a2 = aa[θ2, d2, aa2, α2];
a3 = aa[θ3, d3, aa3, α3];
a4 = aa[θ4, d4, aa4, α4];
a5 = aa[θ5, d5, aa5, α5];
a6 = aa[θ6, d6, aa6, α6];
MatrixForm[a1]
MatrixForm[a2]
MatrixForm[a3]
MatrixForm[a4]
MatrixForm[a5]
MatrixForm[a6]

t6 = a1 . a2 . a3 . a4 . a5 . a6;
MatrixForm[t6]
MatrixForm[t6 /. {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3,
   Cos[θ4] -> c4, Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]

MatrixForm[t6reqd]
MatrixForm[a1]
inva1 = Simplify[Inverse[a1]];
MatrixForm[inva1]
MatrixForm[inva1 . t6reqd /. {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3,
   Cos[θ4] -> c4, Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
inva1t6 = a2 . a3 . a4 . a5 . a6;
MatrixForm[inva1t6]
MatrixForm[inva1t6 /. {Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ2] -> s2, Sin[θ3] -> s3,
   Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
```

$$\begin{pmatrix} \text{Cos}[\theta 1] & 0 & \text{Sin}[\theta 1] & 0 \\ \text{Sin}[\theta 1] & 0 & -\text{Cos}[\theta 1] & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 2] & -\text{Sin}[\theta 2] & 0 & \text{aa2 Cos}[\theta 2] \\ \text{Sin}[\theta 2] & \text{Cos}[\theta 2] & 0 & \text{aa2 Sin}[\theta 2] \\ 0 & 0 & 1 & \text{d2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 3] & 0 & \text{Sin}[\theta 3] & 0 \\ \text{Sin}[\theta 3] & 0 & -\text{Cos}[\theta 3] & 0 \\ 0 & 1 & 0 & \text{d3} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 4] & 0 & -\text{Sin}[\theta 4] & \text{aa4 Cos}[\theta 4] \\ \text{Sin}[\theta 4] & 0 & \text{Cos}[\theta 4] & \text{aa4 Sin}[\theta 4] \\ 0 & -1 & 0 & \text{d4} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 5] & 0 & \text{Sin}[\theta 5] & 0 \\ \text{Sin}[\theta 5] & 0 & -\text{Cos}[\theta 5] & 0 \\ 0 & 1 & 0 & \text{d5} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta 6] & -\text{Sin}[\theta 6] & 0 & \text{aa6 Cos}[\theta 6] \\ \text{Sin}[\theta 6] & \text{Cos}[\theta 6] & 0 & \text{aa6 Sin}[\theta 6] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
( Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] − Cos[θ1] Sin[θ2] Sin[θ3]) )
  Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] − Sin[θ1] Sin[θ2] Sin[θ3]) )
                                                        Cos[θ6] (Cos[θ4] Cos[θ5] (Cos[θ3
```

$$\begin{pmatrix} c6\,(c5\,(c4\,(c1\,c2\,c3 - c1\,s2\,s3) + s1\,s4) + (-c1\,c3\,s2 - c1\,c2\,s3)\,s5) + (c4\,s1 - (c1\,c2 \\ c6\,(c5\,(c4\,(c2\,c3\,s1 - s1\,s2\,s3) - c1\,s4) + (-c3\,s1\,s2 - c2\,s1\,s3)\,s5) + (-c1\,c4 - (c2\,c3 \\ c6\,(c4\,c5\,(c3\,s2 + c2\,s3) + (c2\,c3 - s2\,s3)\,s5) - (c3\,s2 + c2\,s3)\,s4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} nx & ox & ax & px \\ ny & oy & ay & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{Cos}[\theta 1] & 0 & \mathrm{Sin}[\theta 1] & 0 \\ \mathrm{Sin}[\theta 1] & 0 & -\mathrm{Cos}[\theta 1] & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{Cos}[\theta 1] & \mathrm{Sin}[\theta 1] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{Sin}[\theta 1] & -\mathrm{Cos}[\theta 1] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c1\,nx + ny\,s1 & c1\,ox + oy\,s1 & ax\,c1 + ay\,s1 & c1\,px + py\,s1 \\ nz & oz & az & pz \\ -c1\,ny + nx\,s1 & -c1\,oy + ox\,s1 & -ay\,c1 + ax\,s1 & -c1\,py + px\,s1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{Cos}[\theta 6]\,(\mathrm{Cos}[\theta 4]\,\mathrm{Cos}[\theta 5]\,(\mathrm{Cos}[\theta 2]\,\mathrm{Cos}[\theta 3] - \mathrm{Sin}[\theta 2]\,\mathrm{Sin}[\theta 3]) + (-\mathrm{Cos}[\theta 3]\,\mathrm{Sin}[\theta \\ \mathrm{Cos}[\theta 6]\,(\mathrm{Cos}[\theta 4]\,\mathrm{Cos}[\theta 5]\,(\mathrm{Cos}[\theta 3]\,\mathrm{Sin}[\theta 2] + \mathrm{Cos}[\theta 2]\,\mathrm{Sin}[\theta 3]) + (\mathrm{Cos}[\theta 2]\,\mathrm{Cos}[\theta 3 \\ \mathrm{Cos}[\theta 5]\,\mathrm{Cos}[\theta 6]\,\mathrm{Sin} \\ \end{pmatrix}$$

$$\begin{pmatrix} c6\,(c4\,c5\,(c2\,c3 - s2\,s3) + (-c3\,s2 - c2\,s3)\,s5) - (c2\,c3 - s2\,s3)\,s4\,s6 & -c6\,(c2\,c3 - s2 \\ c6\,(c4\,c5\,(c3\,s2 + c2\,s3) + (c2\,c3 - s2\,s3)\,s5) - (c3\,s2 + c2\,s3)\,s4\,s6 & -c6\,(c3\,s2 + \\ c5\,c6\,s4 + c4\,s6 & \\ 0 & \end{pmatrix}$$

```
MatrixForm[a2]
inva2 = Simplify[Inverse[a2]];
MatrixForm[inva2]
MatrixForm[
 inva2 . inva1 . t6reqd /. {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3,
   Cos[θ4] -> c4, Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
inva2a1t6 = a3 . a4 . a5 . a6;
MatrixForm[inva2a1t6]
MatrixForm[inva2a1t6 /. {Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ2] -> s2, Sin[θ3] -> s3,
   Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
```

```
( Cos[θ2]  -Sin[θ2]  0  aa2 Cos[θ2] )
( Sin[θ2]   Cos[θ2]  0  aa2 Sin[θ2] )
(    0         0     1      d2      )
(    0         0     0      1       )
```

```
(  Cos[θ2]  Sin[θ2]  0  -aa2 )
( -Sin[θ2]  Cos[θ2]  0    0  )
(     0        0     1  -d2  )
(     0        0     0    1  )
```

```
( c1 c2 nx + c2 ny s1 + nz s2   c1 c2 ox + c2 oy s1 + oz s2   ax c1 c2 + ay c2 s1 + az s2   -aa2
( c2 nz - c1 nx s2 - ny s1 s2   c2 oz - c1 ox s2 - oy s1 s2   az c2 - ax c1 s2 - ay s1 s2
(       -c1 ny + nx s1                -c1 oy + ox s1                -ay c1 + ax s1
(              0                             0                             0
```

```
( Cos[θ6] (Cos[θ3] Cos[θ4] Cos[θ5] - Sin[θ3] Sin[θ5]) - Cos[θ3] Sin[θ4] Sin[θ6]
( Cos[θ6] (Cos[θ4] Cos[θ5] Sin[θ3] + Cos[θ3] Sin[θ5]) - Sin[θ3] Sin[θ4] Sin[θ6]
(                  Cos[θ5] Cos[θ6] Sin[θ4] + Cos[θ4] Sin[θ6]
(                                      0
```

```
( c6 (c3 c4 c5 - s3 s5) - c3 s4 s6   -c3 c6 s4 - (c3 c4 c5 - s3 s5) s6   c5 s3 + c3 c4 s5
( c6 (c4 c5 s3 + c3 s5) - s3 s4 s6   -c6 s3 s4 - (c4 c5 s3 + c3 s5) s6   -c3 c5 + c4 s3 s5
(         c5 c6 s4 + c4 s6                     c4 c6 - c5 s4 s6                  s4 s5
(                0                                    0                            0
```

```
MatrixForm[a3]
inva3 = Simplify[Inverse[a3]];
MatrixForm[inva3]
MatrixForm[inva3 . inva2 . inva1 . t6reqd /.
  {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
inva3a2a1t6 = a4 . a5 . a6;
MatrixForm[inva3a2a1t6]
MatrixForm[inva3a2a1t6 /. {Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ2] -> s2, Sin[θ3] -> s3,
   Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
```

$$\begin{pmatrix} Cos[\theta3] & 0 & Sin[\theta3] & 0 \\ Sin[\theta3] & 0 & -Cos[\theta3] & 0 \\ 0 & 1 & 0 & d3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} Cos[\theta3] & Sin[\theta3] & 0 & 0 \\ 0 & 0 & 1 & -d3 \\ Sin[\theta3] & -Cos[\theta3] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} nz\,(c3\,s2+c2\,s3)+c1\,nx\,(c2\,c3-s2\,s3)+ny\,s1\,(c2\,c3-s2\,s3) & oz\,(c3\,s2+c2\,s3) \\ -c1\,ny+nx\,s1 & \\ c1\,nx\,(c3\,s2+c2\,s3)+ny\,s1\,(c3\,s2+c2\,s3)+nz\,(-c2\,c3+s2\,s3) & c1\,ox\,(c3\,s2+c2\,s3 \\ 0 & \end{pmatrix}$$

$$\begin{pmatrix} Cos[\theta4]\,Cos[\theta5]\,Cos[\theta6]-Sin[\theta4]\,Sin[\theta6] & -Cos[\theta6]\,Sin[\theta4]-Cos[\theta4]\,Cos[\theta5] \\ Cos[\theta5]\,Cos[\theta6]\,Sin[\theta4]+Cos[\theta4]\,Sin[\theta6] & Cos[\theta4]\,Cos[\theta6]-Cos[\theta5]\,Sin[\theta4]\,Sin \\ -Cos[\theta6]\,Sin[\theta5] & Sin[\theta5]\,Sin[\theta6] \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} c4\,c5\,c6-s4\,s6 & -c6\,s4-c4\,c5\,s6 & c4\,s5 & aa4\,c4+aa6\,c4\,c5\,c6-d5\,s4-aa6\,s4\,s6 \\ c5\,c6\,s4+c4\,s6 & c4\,c6-c5\,s4\,s6 & s4\,s5 & c4\,d5+aa4\,s4+aa6\,c5\,c6\,s4+aa6\,c4\,s6 \\ -c6\,s5 & s5\,s6 & c5 & d4-aa6\,c6\,s5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
MatrixForm[a4]
inva4 = Simplify[Inverse[a4]];
MatrixForm[inva4]
MatrixForm[inva4 . inva3 . inva2 . inva1 . t6reqd /.
  {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
inva4a3a2a1t6 = a5 . a6;
MatrixForm[inva4a3a2a1t6]
MatrixForm[inva4a3a2a1t6 /. {Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ2] -> s2, Sin[θ3] -> s3,
   Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
```

$$\begin{pmatrix} \text{Cos}[\theta4] & 0 & -\text{Sin}[\theta4] & \text{aa4 Cos}[\theta4] \\ \text{Sin}[\theta4] & 0 & \text{Cos}[\theta4] & \text{aa4 Sin}[\theta4] \\ 0 & -1 & 0 & \text{d4} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[\theta4] & \text{Sin}[\theta4] & 0 & -\text{aa4} \\ 0 & 0 & -1 & \text{d4} \\ -\text{Sin}[\theta4] & \text{Cos}[\theta4] & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
( nz (c3 c4 s2 + c2 c4 s3) + ny (s1 (c2 c3 c4 - c4 s2 s3) - c1 s4) + nx (c1 (c2 c3 c4 -
                    c1 nx (-c3 s2 - c2 s3) + ny s1 (-c3 s2 - c2 s3) + nz (c2 c3 - s2 s3)
  nz (-c3 s2 s4 - c2 s3 s4) + nx (c4 s1 + c1 (-c2 c3 s4 + s2 s3 s4)) + ny (-c1 c4 + s1 (-c2
                                                      0
```

$$\begin{pmatrix} \text{Cos}[\theta5]\,\text{Cos}[\theta6] & -\text{Cos}[\theta5]\,\text{Sin}[\theta6] & \text{Sin}[\theta5] & \text{aa6 Cos}[\theta5]\,\text{Cos}[\theta6] \\ \text{Cos}[\theta6]\,\text{Sin}[\theta5] & -\text{Sin}[\theta5]\,\text{Sin}[\theta6] & -\text{Cos}[\theta5] & \text{aa6 Cos}[\theta6]\,\text{Sin}[\theta5] \\ \text{Sin}[\theta6] & \text{Cos}[\theta6] & 0 & \text{d5} + \text{aa6 Sin}[\theta6] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c5\,c6 & -c5\,s6 & s5 & aa6\,c5\,c6 \\ c6\,s5 & -s5\,s6 & -c5 & aa6\,c6\,s5 \\ s6 & c6 & 0 & d5 + aa6\,s6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
MatrixForm[a5]
inva5 = Simplify[Inverse[a5]];
MatrixForm[inva5]
MatrixForm[inva4 . inva4 . inva3 . inva2 . inva1 . t6reqd /.
  {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
inva5a4a3a2a1t6 = a6;
MatrixForm[inva5a4a3a2a1t6]
MatrixForm[inva5a4a3a2a1t6 /. {Cos[θ2] -> c2, Cos[θ3] -> c3,
   Cos[θ4] -> c4, Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6}]
```

```
( Cos[θ5]  0   Sin[θ5]   0 )
( Sin[θ5]  0  -Cos[θ5]   0 )
(    0     1     0      d5 )
(    0     0     0       1 )
```

```
( Cos[θ5]   Sin[θ5]   0   0  )
(    0         0      1  -d5 )
( Sin[θ5]  -Cos[θ5]   0   0  )
(    0         0      0   1  )
```

```
(     nz (c2 (c4^2 s3 + c3 s4) + s2 (c3 c4^2 - s3 s4)) + nx (c4 s1 s4 + c1 (-s2 (c4^2 s3 + c3
                                                nz (c3 s2 s4 + c2 s3 s4) + nx (-c4 s1 + c1 (c2
( nz (s2 (-c4 s3 - c3 c4 s4) + c2 (c3 c4 - c4 s3 s4)) + nx (-s1 s4^2 + c1 (c2 (-c4 s3 - c3
(
```

```
( Cos[θ6]  -Sin[θ6]  0  aa6 Cos[θ6] )
( Sin[θ6]   Cos[θ6]  0  aa6 Sin[θ6] )
(    0         0     1       0      )
(    0         0     0       1      )
```

```
( c6  -s6  0  aa6 c6 )
( s6   c6  0  aa6 s6 )
(  0    0  1    0    )
(  0    0  0    1    )
```

```
MatrixForm[t6]
t6[[1, 1]]
Simplify[t6[[1, 1]]]
```

```
( Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Sin[θ2] Sin[θ3])
  Cos[θ6] (Cos[θ5] (Cos[θ4] (Cos[θ2] Cos[θ3] Sin[θ1] - Sin[θ1] Sin[θ2] Sin[θ3])
                                                   Cos[θ6] (Cos[θ4] Cos[θ5] (Cos[θ3
(
```

```
Cos[θ6]
   (Cos[θ5] (Cos[θ4] (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Sin[θ2] Sin[θ3]) +
        Sin[θ1] Sin[θ4]) +
     (-Cos[θ1] Cos[θ3] Sin[θ2] - Cos[θ1] Cos[θ2] Sin[θ3]) Sin[θ5]) +
 (Cos[θ4] Sin[θ1] -
     (Cos[θ1] Cos[θ2] Cos[θ3] - Cos[θ1] Sin[θ2] Sin[θ3]) Sin[θ4])
  Sin[θ6]
```

```
Sin[θ1] (Cos[θ5] Cos[θ6] Sin[θ4] + Cos[θ4] Sin[θ6]) +
 Cos[θ1] (-Cos[θ6] Sin[θ2 + θ3] Sin[θ5] +
     Cos[θ2 + θ3] (Cos[θ4] Cos[θ5] Cos[θ6] - Sin[θ4] Sin[θ6]))
```

■ Frame Trig Simplifications

```
baseXFMframe6 = Simplify[a1 . a2 . a3 . a4 . a5 . a6] /.
  {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6};
baseXFMframe6 // MatrixForm
```

```
(         s1 (c5 c6 s4 + c4 s6) + c1 ((c4 c5 c6 - s4 s6) Cos[θ2 + θ3] - c6 s5 Sin[θ2 + θ
  -s6 (c1 c4 + s1 s4 Cos[θ2 + θ3]) + c6 (c5 (c2 c3 c4 s1 - c4 s1 s2 s3 - c1 s4) - s1 s5 Sin
                c6 s5 Cos[θ2 + θ3] + c4 c5 c6 Sin[θ2 + θ3] - s4 s6 Sin[θ2 + θ3]
(                                                  0
```

```
baseXFMframe2 = Simplify[a1 . a2] /.
  {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6};
baseXFMframe2 // MatrixForm
```

$$\begin{pmatrix} c1\,c2 & -c1\,s2 & s1 & aa2\,c1\,c2 + d2\,s1 \\ c2\,s1 & -s1\,s2 & -c1 & -c1\,d2 + aa2\,c2\,s1 \\ s2 & c2 & 0 & aa2\,s2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
baseXFMframe3 = Simplify[Simplify[a1 . a2] . a3] /.
  {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6};
baseXFMframe3 // MatrixForm
```

$$\begin{pmatrix} c1\,\mathrm{Cos}[\theta 2 + \theta 3] & s1 & c1\,\mathrm{Sin}[\theta 2 + \theta 3] & aa2\,c1\,c2 + (d2 + d3)\,s1 \\ s1\,\mathrm{Cos}[\theta 2 + \theta 3] & -c1 & s1\,\mathrm{Sin}[\theta 2 + \theta 3] & -c1\,(d2 + d3) + aa2\,c2\,s1 \\ \mathrm{Sin}[\theta 2 + \theta 3] & 0 & -\mathrm{Cos}[\theta 2 + \theta 3] & aa2\,s2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
baseXFMframe4 = Simplify[Simplify[Simplify[a1 . a2] . a3] . a4];
baseXFMframe4 /. {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3,
   Cos[θ4] -> c4, Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6} //
 MatrixForm
```

$$\begin{pmatrix} s1\,s4 + c1\,c4\,\mathrm{Cos}[\theta 2 + \theta 3] & -c1\,\mathrm{Sin}[\theta 2 + \theta 3] & c4\,s1 - c1\,s4\,\mathrm{Cos}[\theta 2 + \theta 3] & s1\,(d2 + \\ -c1\,s4 + c4\,s1\,\mathrm{Cos}[\theta 2 + \theta 3] & -s1\,\mathrm{Sin}[\theta 2 + \theta 3] & -c1\,c4 - s1\,s4\,\mathrm{Cos}[\theta 2 + \theta 3] & -c1\,(d2 + \\ c4\,\mathrm{Sin}[\theta 2 + \theta 3] & \mathrm{Cos}[\theta 2 + \theta 3] & -s4\,\mathrm{Sin}[\theta 2 + \theta 3] & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
baseXFMframe44 = TrigReduce[TrigReduce[TrigReduce[a1 . a2] . a3] . a4];
baseXFMframe44 // MatrixForm
```

```
( 1/4 (2 Cos[θ1 - θ4] + Cos[θ1 - θ2 - θ3 - θ4] + Cos[θ1 + θ2 + θ3 - θ4] - 2 Cos[θ1 + θ4] + Cos
  1/4 (2 Sin[θ1 - θ4] + Sin[θ1 - θ2 - θ3 - θ4] + Sin[θ1 + θ2 + θ3 - θ4] - 2 Sin[θ1 + θ4] + Sin
                                      1/2 (Sin[θ2 + θ3 - θ4] + Sin[θ2 + θ3 + θ4]
                                                  0
```

```
baseXFMframe44 = TrigExpand[TrigExpand[TrigExpand[a1 . a2] . a3] . a4];
baseXFMframe44 /. {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3,
   Cos[θ4] -> c4, Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6} //
 MatrixForm
```

```
( c1 c2 c3 c4 - c1 c4 s2 s3 + s1 s4   -c1 c3 s2 - c1 c2 s3   c4 s1 - c1 c2 c3 s4 + c1 s2 s3 s4
  c2 c3 c4 s1 - c4 s1 s2 s3 - c1 s4   -c3 s1 s2 - c2 s1 s3   -c1 c4 - c2 c3 s1 s4 + s1 s2 s3
        c3 c4 s2 + c2 c4 s3                c2 c3 - s2 s3           -c3 s2 s4 - c2 s3 s4
                 0                               0                          0
```

```
baseXFMframe44 = TrigFactor[TrigFactor[TrigFactor[a1 . a2] . a3] . a4];
baseXFMframe44 /. {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3,
   Cos[θ4] -> c4, Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6} //
 MatrixForm
```

```
( 1/4 (2 Cos[θ1 - θ4] + Cos[θ1 - θ2 - θ3 - θ4] + Cos[θ1 + θ2 + θ3 - θ4] - 2 Cos[θ1 + θ4] + Cos
  1/4 (2 Sin[θ1 - θ4] + Sin[θ1 - θ2 - θ3 - θ4] + Sin[θ1 + θ2 + θ3 - θ4] - 2 Sin[θ1 + θ4] + Sin
                                                   c4 Sin[θ2 + θ3]
                                                  0
```

```
baseXFMframe5 =
 Simplify[Simplify[Simplify[Simplify[a1 . a2] . a3] . a4] . a5] /.
  {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3, Cos[θ4] -> c4,
   Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6};
baseXFMframe5 // MatrixForm
```

```
( c5 s1 s4 + c1 (c4 c5 Cos[θ2 + θ3] - s5 Sin[θ2 + θ3])     c4 s1 - c1 s4 Cos[θ2 + θ3]    s1
 -c1 c5 s4 + c4 c5 s1 Cos[θ2 + θ3] - s1 s5 Sin[θ2 + θ3]   -c1 c4 - s1 s4 Cos[θ2 + θ3]   s5
        s5 Cos[θ2 + θ3] + c4 c5 Sin[θ2 + θ3]                    -s4 Sin[θ2 + θ3]
                          0                                            0
```

```
baseXFMframe6 = Simplify[
  Simplify[Simplify[Simplify[Simplify[a1 . a2] . a3] . a4] . a5] . a6];
baseXFMframe6 /. {Cos[θ1] -> c1, Cos[θ2] -> c2, Cos[θ3] -> c3,
   Cos[θ4] -> c4, Cos[θ5] -> c5, Cos[θ6] -> c6, Sin[θ1] -> s1, Sin[θ2] -> s2,
   Sin[θ3] -> s3, Sin[θ4] -> s4, Sin[θ5] -> s5, Sin[θ6] -> s6} // MatrixForm
baseXFMframe6 // MatrixForm
```

```
( s1 (c5 c6 s4 + c4 s6) + c1 ((c4 c5 c6 - s4 s6) Cos[θ2 + θ3] - c6 s5 Sin[θ2 + θ3])   c6
 -c1 (c5 c6 s4 + c4 s6) + s1 (c4 c5 c6 - s4 s6) Cos[θ2 + θ3] - c6 s1 s5 Sin[θ2 + θ3]
        c6 s5 Cos[θ2 + θ3] + c4 c5 c6 Sin[θ2 + θ3] - s4 s6 Sin[θ2 + θ3]
                                   0
```

```
(  Sin[θ1] (Cos[θ5] Cos[θ6] Sin[θ4] + Cos[θ4] Sin[θ6]) + Cos[θ1] (-Cos[θ6] Sin
 -Cos[θ6] Sin[θ1] Sin[θ2 + θ3] Sin[θ5] - Cos[θ1] (Cos[θ5] Cos[θ6] Sin[θ4] + Cos[
                                  Cos[θ4] Cos[θ5] Cos[θ6] Sin[θ2 + θ3] + Cos[θ2 + θ3
                                                                                0
```

```
baseXFMframe6 /.
  {θ1 -> (-60 * 0 * Pi / 180 + θ1n), θ2 -> (4.123 * 1 * Pi / 180 + θ2n),
   θ3 -> (-4.123 * 1 * Pi / 180 + θ3n), θ4 -> (-60 * 0 * Pi / 180 + θ4n),
   θ5 -> (θ5n - 1 * 90 * Pi / 180), θ6 -> (0 * Pi / 180 + θ6n)} //
 MatrixForm
```

```
( 0   0  1             -aa4 + d2 + d3            )
( 1.  0. 0  0.997412 aa2 + 1. aa6 + 0. d4 + 1. d5 )
( 0.  1. 0  0.0718978 aa2 - 1. d4 + 0. (aa6 + d5) )
( 0   0  0                   1                   )
```

■ Positioning of Frame Four

The positioning and orientation control over frame four will be studied in this section. First, see if the position of frame four may be arbitrarily controlled along the minus Y axis (yb) of the base frame which is centered on the aperture while maintaining the frame's nominal orientation. Since there's a fixed transformation relating the zero frame to the base frame, this analysis will be carried out using frame zero.

```
baseXFMframe4 // MatrixForm
baseXFMframe4 /. {θ1 -> (-60 * 0 * Pi / 180 + θ1n),
   θ2 -> (4.123 * 1 * Pi / 180 + θ2n), θ3 -> (-4.123 * 1 * Pi / 180 + θ3n),
   θ4 -> (-60 * 0 * Pi / 180 + θ4n)} // MatrixForm
t4reqd = {{-1, 0, 0, px}, {0, 0, 1, py}, {0, 1, 0, pz}, {0, 0, 0, 1}};
t4reqd // MatrixForm
```

```
( Cos[θ1] Cos[θ2 + θ3] Cos[θ4] + Sin[θ1] Sin[θ4]   -Cos[θ1] Sin[θ2 + θ3]   Cos[θ4]
( Cos[θ2 + θ3] Cos[θ4] Sin[θ1] - Cos[θ1] Sin[θ4]   -Sin[θ1] Sin[θ2 + θ3]   -Cos[θ1]
(              Cos[θ4] Sin[θ2 + θ3]                      Cos[θ2 + θ3]
(                       0                                     0
```

```
( -1  0   0        -aa4 + d2 + d3     )
(  0  0.  1.   0.997412 aa2 + 0. d4   )
(  0  1.  0.  0.0718978 aa2 - 1. d4   )
(  0  0   0              1            )
```

```
( -1  0  0  px )
(  0  0  1  py )
(  0  1  0  pz )
(  0  0  0  1  )
```

Element 3,2 of t4reqd implies theta2+theta3 = a multiple integral of 2 Pi thus

```
baseXFMframe4 /. {θ3 -> -θ2} // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\theta 1]\,\text{Cos}[\theta 4]+\text{Sin}[\theta 1]\,\text{Sin}[\theta 4] & 0 & \text{Cos}[\theta 4]\,\text{Sin}[\theta 1]-\text{Cos}[\theta 1]\,\text{Sin}[\theta 4] & \text{Cos}[\\ \text{Cos}[\theta 4]\,\text{Sin}[\theta 1]-\text{Cos}[\theta 1]\,\text{Sin}[\theta 4] & 0 & -\text{Cos}[\theta 1]\,\text{Cos}[\theta 4]-\text{Sin}[\theta 1]\,\text{Sin}[\theta 4] & (\text{aa2} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

Element 3,4 of t4reqd yields the following

```
stwo = Solve[baseXFMframe4[[3, 4]] == t4reqd[[3, 4]] /. {θ2 + θ3 -> 0},
    θ2][[1]][[1]]
```

```
Solve::ifun :
 Inverse functions are being used by Solve, so some solutions may not be found.
```

$$\theta 2 \to \text{ArcSin}\left[\frac{\text{d4}+\text{pz}}{\text{aa2}}\right]$$

```
Simplify[Simplify[baseXFMframe4 /. {θ3 -> -θ2}] /. stwo] // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\theta 1-\theta 4] & 0 & \text{Sin}[\theta 1-\theta 4] & \text{Cos}[\theta 1]\left(\text{aa2}\sqrt{1-\frac{(\text{d4}+\text{pz})^2}{\text{aa2}^2}}+\text{aa4}\,\text{Cos}[\theta 4]\right)+\text{Sin}[\theta 1 \\ \text{Sin}[\theta 1-\theta 4] & 0 & -\text{Cos}[\theta 1-\theta 4] & \left(\text{aa2}\sqrt{1-\frac{(\text{d4}+\text{pz})^2}{\text{aa2}^2}}+\text{aa4}\,\text{Cos}[\theta 4]\right)\text{Sin}[\theta 1]-\text{Cos}[\theta 1 \\ 0 & 1 & 0 & \text{pz} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Element 1,1 of t4reqd yields the following

```
temp = Simplify[Expand[Simplify[
     Simplify[baseXFMframe4 /. {θ3 -> -θ2}] /. stwo] /. {θ1 -> Pi + θ4}]];
temp // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -\text{aa4}-\text{aa2}\sqrt{\frac{\text{aa2}^2-(\text{d4}+\text{pz})^2}{\text{aa2}^2}}\,\text{Cos}[\theta 4]-(\text{d2}+\text{d3})\,\text{Sin}[\theta 4] \\ 0 & 0 & 1 & (\text{d2}+\text{d3})\,\text{Cos}[\theta 4]-\text{aa2}\sqrt{\frac{\text{aa2}^2-(\text{d4}+\text{pz})^2}{\text{aa2}^2}}\,\text{Sin}[\theta 4] \\ 0 & 1 & 0 & \text{pz} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
temp /. {aa2 -> 62.6, aa4 -> 46,
     aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
   {θ4 -> -Pi / 2} /. {pz -> 0} //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -21.4 \\ 0 & 0 & 1 & 62.438 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Note that the following solution transform shows that the origin of frame four can not be placed relative to the zero (and therefore base) frame while maintaining the nominal orientation of frame four.

```
solnframe4 =
 temp /. {√((aa2^2 - (d4 + pz)^2) / aa2^2) -> √(aa2^2 - (d4 + pz)^2) / aa2};
solnframe4 // MatrixForm
aperXFMframe0 . solnframe4 // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -aa4 - \sqrt{aa2^2 - (d4+pz)^2}\,Cos[\theta 4] - (d2+d3)\,Sin[\theta 4] \\ 0 & 0 & 1 & (d2+d3)\,Cos[\theta 4] - \sqrt{aa2^2 - (d4+pz)^2}\,Sin[\theta 4] \\ 0 & 1 & 0 & pz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & -21.4 - 1.\left(-aa4 - \sqrt{aa2^2 - (d4+pz)^2}\,Cos[\theta 4] - (d2+d3)\,Sin[\theta 4]\right) \\ 0. & 0. & -1. & -1.\left((d2+d3)\,Cos[\theta 4] - \sqrt{aa2^2 - (d4+pz)^2}\,Sin[\theta 4]\right) \\ 0. & 1. & 0. & 1.\,pz \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

```
solnframe4 /. {aa2 -> 62.6, aa4 -> 46,
     aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
   {θ4 -> -Pi / 2} /. {pz -> 0} //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -21.4 \\ 0 & 0 & 1 & 62.438 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Clear[px]
solnframe4[[1, 4]]
t4reqd[[1, 4]]
theta4soln = Solve[solnframe4[[1, 4]] == t4reqd[[1, 4]], θ4]
```

$$-aa4 - \sqrt{aa2^2 - (d4 + pz)^2}\, Cos[\theta 4] - (d2 + d3)\, Sin[\theta 4]$$

```
px
```

Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found.

$$\Big\{\Big\{\theta 4 \to -\mathrm{ArcCos}\Big[\Big(-2\sqrt{}\,(aa2^2\,d2^2 - aa4^2\,d2^2 + d2^4 + 2\,aa2^2\,d2\,d3 - 2\,aa4^2\,d2\,d3 + 4\,d2^3\,d3 + aa2^2\,d3^2 - aa4^2\,d3^2 + 6\,d2^2\,d3^2 + 4\,d2\,d3^3 + d3^4 - d2^2\,d4^2 - 2\,d2\,d3\,d4^2 - d3^2\,d4^2 - 2\,aa4\,d2^2\,px - 4\,aa4\,d2\,d3\,px - 2\,aa4\,d3^2\,px - d2^2\,px^2 - 2\,d2\,d3\,px^2 - d3^2\,px^2 - 2\,d2^2\,d4\,pz - 4\,d2\,d3\,d4\,pz - 2\,d3^2\,d4\,pz - d2^2\,pz^2 - 2\,d2\,d3\,pz^2 - d3^2\,pz^2) - (2\,aa4 + 2\,px)\sqrt{aa2^2 - (d4 + pz)^2}\Big) \Big/ (2\,(aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - 2\,d4\,pz - pz^2))\Big]\Big\},$$

$$\Big\{\theta 4 \to \mathrm{ArcCos}\Big[\Big(-2\sqrt{}\,(aa2^2\,d2^2 - aa4^2\,d2^2 + d2^4 + 2\,aa2^2\,d2\,d3 - 2\,aa4^2\,d2\,d3 + 4\,d2^3\,d3 + aa2^2\,d3^2 - aa4^2\,d3^2 + 6\,d2^2\,d3^2 + 4\,d2\,d3^3 + d3^4 - d2^2\,d4^2 - 2\,d2\,d3\,d4^2 - d3^2\,d4^2 - 2\,aa4\,d2^2\,px - 4\,aa4\,d2\,d3\,px - 2\,aa4\,d3^2\,px - d2^2\,px^2 - 2\,d2\,d3\,px^2 - d3^2\,px^2 - 2\,d2^2\,d4\,pz - 4\,d2\,d3\,d4\,pz - 2\,d3^2\,d4\,pz - d2^2\,pz^2 - 2\,d2\,d3\,pz^2 - d3^2\,pz^2) - (2\,aa4 + 2\,px)\sqrt{aa2^2 - (d4 + pz)^2}\Big) \Big/ (2\,(aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - 2\,d4\,pz - pz^2))\Big]\Big\},$$

$$\Big\{\theta 4 \to -\mathrm{ArcCos}\Big[\Big(2\sqrt{}\,(aa2^2\,d2^2 - aa4^2\,d2^2 + d2^4 + 2\,aa2^2\,d2\,d3 - 2\,aa4^2\,d2\,d3 + 4\,d2^3\,d3 + aa2^2\,d3^2 - aa4^2\,d3^2 + 6\,d2^2\,d3^2 + 4\,d2\,d3^3 + d3^4 - d2^2\,d4^2 - 2\,d2\,d3\,d4^2 - d3^2\,d4^2 - 2\,aa4\,d2^2\,px - 4\,aa4\,d2\,d3\,px - 2\,aa4\,d3^2\,px - d2^2\,px^2 - 2\,d2\,d3\,px^2 - d3^2\,px^2 - 2\,d2^2\,d4\,pz - 4\,d2\,d3\,d4\,pz - 2\,d3^2\,d4\,pz - d2^2\,pz^2 - 2\,d2\,d3\,pz^2 - d3^2\,pz^2) - (2\,aa4 + 2\,px)\sqrt{aa2^2 - (d4 + pz)^2}\Big) \Big/ (2\,(aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - 2\,d4\,pz - pz^2))\Big]\Big\},$$

$$\Big\{\theta 4 \to \mathrm{ArcCos}\Big[\Big(2\sqrt{}\,(aa2^2\,d2^2 - aa4^2\,d2^2 + d2^4 + 2\,aa2^2\,d2\,d3 - 2\,aa4^2\,d2\,d3 + 4\,d2^3\,d3 + aa2^2\,d3^2 - aa4^2\,d3^2 + 6\,d2^2\,d3^2 + 4\,d2\,d3^3 + d3^4 - d2^2\,d4^2 - 2\,d2\,d3\,d4^2 - d3^2\,d4^2 - 2\,aa4\,d2^2\,px - 4\,aa4\,d2\,d3\,px - 2\,aa4\,d3^2\,px - d2^2\,px^2 - 2\,d2\,d3\,px^2 - d3^2\,px^2 - 2\,d2^2\,d4\,pz - 4\,d2\,d3\,d4\,pz - 2\,d3^2\,d4\,pz - d2^2\,pz^2 - 2\,d2\,d3\,pz^2 - d3^2\,pz^2) - (2\,aa4 + 2\,px)\sqrt{aa2^2 - (d4 + pz)^2}\Big) \Big/ (2\,(aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - 2\,d4\,pz - pz^2))\Big]\Big\}\Big\}$$

```
Clear[px]
theta4soln /. {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1, d2 -> 6.9,
    d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. {pz -> 0} /. {px -> -21.4}
theta4soln[[3]][[1]] /. {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
    d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. {pz -> 0} /.
 {px -> -21.4}
```

$\{\{\theta 4 \to -2.32143\}, \{\theta 4 \to 2.32143\}, \{\theta 4 \to -1.5708\}, \{\theta 4 \to 1.5708\}\}$

$\theta 4 \to -1.5708$

```
Simplify[solnframe4 /. theta4soln[[3]][[1]]] // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -aa4 + \frac{\sqrt{aa2^2-(d4+pz)^2}\left(-\sqrt{(d2+d3)^2\,(aa2^2-aa4^2+d2^2+2\,d2\,d3+d3^2-d4^2-2\,aa4\,px-px^2-2\,d4\,pz-pz^2)}+aa4\sqrt{\ldots}\right.}{aa2^2+d2^2+2\,d2\,d3+d3^2-d4^2-2\,d4\,pz-pz^2} \\ 0 & 0 & 1 & \frac{(d2+d3)\left(\sqrt{(d2+d3)^2\,(aa2^2-aa4^2+d2^2+2\,d2\,d3+d3^2-d4^2-2\,aa4\,px-px^2-2\,d4\,pz-pz^2)}-aa4\sqrt{aa2^2-(d4+pz)^2}\right.}{aa2^2+d2^2+2\,d2\,d3+d3^2-d4^2-2\,d4\,pz-pz^2} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[solnframe4 /. {theta4soln[[3]][[1]]} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -46 + \frac{-179331.+414.\,pz+46.\,pz^2+1.\,px\,(-58.1+pz)\,(67.1+pz)+1.\,\sqrt{-1.\,(-58.1+pz)\,(67.1+pz)}\,\sqrt{-55674.7\ldots}}{(-62.7601+pz)\,(71.7601+pz)} \\ 0 & 0 & 1 & -\frac{12.3\left(-2\,(46+px)\,\sqrt{-1.\,(-58.1+pz)\,(67.1+pz)}+2\sqrt{-55674.7\,px-605.16\,px^2-605.16\,(-44.5706+pz)\,(53.5706\ldots}\right.}{(-62.7601+pz)\,(71.7601+pz)} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[aperXFMframe0 . solnframe4 . a5 . a6 . frame6XFMhotspot /.
   theta4soln[[3]][[1]]] // MatrixForm
Simplify[% /. {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
    d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] // MatrixForm
Simplify[% /. {pz -> 0}] // MatrixForm
Simplify[aperXFMframe0 . solnframe4 .
      a5 . a6 . frame6XFMhotspot /. theta4soln[[3]][[1]] /.
    {pz -> 0, px -> -aa4 + d2 + d3} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
% /. {θ5 -> θ5n - Pi / 2, θ6 -> θ6n} // MatrixForm
Simplify[aperXFMframe0 . solnframe4 .
    a5 . a6 . frame6XFMhotspot /. theta4soln[[3]][[1]]] //
 MatrixForm
```

```
0. Cos[θ5] - 1. Sin[θ5]   -1. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]

0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 1. Sin[θ6]   -

1. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] - 1. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
```

```
0. Cos[θ5] - 1. Sin[θ5]   -1. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]

0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 1. Sin[θ6]   -

1. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] - 1. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
```

```
⎛
⎜ 0. Cos[θ5] - 1. Sin[θ5]  -1. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
⎜
⎜ 0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 1. Sin[θ6]   -
⎜
⎜ 1. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] - 1. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
⎝ 0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
```

```
⎛ 0. Cos[θ5] - 1. Sin[θ5]  -1. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
⎜ 0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 1. Sin[θ6]   -
⎜ 1. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] - 1. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
⎝ 0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
```

```
⎛ 1.  0.  0.     0.    ⎞
⎜ 0.  1.  0.  -105.238 ⎟
⎜ 0.  0.  1.     0.    ⎟
⎝ 0.  0.  0.     1.    ⎠
```

```
⎛
⎜ 0. Cos[θ5] - 1. Sin[θ5]  -1. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
⎜
⎜ 0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 1. Sin[θ6]   -
⎜
⎜ 1. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] - 1. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
⎝ 0. Cos[θ5] + 0. Sin[θ5]   0. Cos[θ5] Cos[θ6] + 0. Cos[θ6] Sin[θ5] + 0. Sin[θ6]
```

Conclusion: Only 2 DOF of the origin relative to the zero (and hence base) frames are free to be chosen the third DOF (either x or y) will be fixed after defining these two. In the above derivation, selecting pz and px defines the frame four origin wrt frame zero. The following shows various evaluations.

```
Simplify[solnframe4 /. {theta4soln[[3]][[1]]} /. {px -> 0, pz -> 0}] //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -aa4 + \frac{aa2^2\ aa4 - aa4\ d4^2 - \sqrt{aa2^2 - d4^2}\ \sqrt{(d2+d3)^2\ (aa2^2 - aa4^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2)}}{aa2^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2} + (d2 + d3) \sqrt{-\ \cdots} \\ 0 & 0 & 1 & \frac{(d2+d3)\ \left(-aa4 \sqrt{aa2^2 - d4^2} + \sqrt{(d2+d3)^2\ (aa2^2 - aa4^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2)}\right)}{aa2^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2} + \sqrt{aa2^2 - d4^2} \sqrt{\frac{aa2 \cdots}{\cdots}} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[solnframe4 /. {theta4soln[[3]][[1]]} /. {pz -> 0}] // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -aa4 - \frac{aa4\ d4^2 + d4^2\ px - aa2^2\ (aa4 + px) + \sqrt{aa2^2 - d4^2}\ \sqrt{(d2+d3)^2\ (aa2^2 - aa4^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2 - 2\ aa4\ px - \cdots}}{aa2^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2} \\ 0 & 0 & 1 & \frac{(d2+d3)\ \left(-aa4 \sqrt{aa2^2 - d4^2} - \sqrt{aa2^2 - d4^2}\ px + \sqrt{(d2+d3)^2\ (aa2^2 - aa4^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2 - 2\ aa4\ px - px^2)}\right)}{aa2^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[
  solnframe4 /. {theta4soln[[3]][[1]]} /. {px -> -aa4 + d2 + d3, pz -> 0}] //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -aa4 - \frac{-aa2^2\ (d2+d3) + d2\ d4^2 + d3\ d4^2 + \sqrt{aa2^2 - d4^2}\ \sqrt{(d2+d3)^2\ (aa2^2 - d4^2)}}{aa2^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2} + (d2 + d3) \sqrt{\frac{aa2^4 + d2^4 + \cdots}{\cdots}} \\ 0 & 0 & 1 & -\frac{(d2+d3)\ \left(d2 \sqrt{aa2^2 - d4^2} + d3 \sqrt{aa2^2 - d4^2} - \sqrt{(d2+d3)^2\ (aa2^2 - d4^2)}\right)}{aa2^2 + d2^2 + 2\ d2\ d3 + d3^2 - d4^2} + \sqrt{aa2^2 - d4^2} \sqrt{\frac{aa2^4 + d2^4 + 4\ d2 \cdots}{\cdots}} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[solnframe4 /. {theta4soln[[3]][[1]]} /.
    {px -> -aa4 + d2 + d3, pz -> 0}] /. {aa2 -> 62.6, aa4 -> 46,
   aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -21.4 \\ 0 & 0 & 1 & 62.438 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Simplify[solnframe4 /. {theta4soln[[3]][[1]]} /. {aa2 -> 62.6, aa4 -> 46,
     aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. {pz -> 0}] //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -6.18104 + 0.86563\,px - 0.34105\sqrt{(21.1094 - px)\,(113.109 + px)} + 24.6 \\ 0 & 0 & 1 & -15.6883 - 0.34105\,px + 0.13437\sqrt{(21.1094 - px)\,(113.109 + px)} + 62.438 \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[solnframe4 /. {theta4soln[[3]][[1]]} /. {aa2 -> 62.6, aa4 -> 46,
     aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
   {px -> -21.4000000000000012`, pz -> 0.000804593299969624808`}] //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -21.4 \\ 0 & 0 & 1 & 62.438 \\ 0 & 1 & 0 & 0.000804593 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Simplify[solnframe4 /. {theta4soln[[3]][[1]]} /. {aa2 -> 62.6, aa4 -> 46,
     aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
   {px -> -21.4, pz -> 0}] //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -21.4 \\ 0 & 0 & 1 & 62.438 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
theta4soln /. {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1, d2 -> 6.9,
    d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. {pz -> 0} /. {px -> -21.4}
N[θ4 * 180 / Pi /. theta4soln /. {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
     d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. {pz -> 0} /.
  {px -> -21.4}]
```

```
{{θ4 → -2.32143}, {θ4 → 2.32143}, {θ4 → -1.5708}, {θ4 → 1.5708}}
```

```
{-133.008, 133.008, -90., 90.}
```

```
baseXFMframe4 /. {θ1 -> (-60 * 0 * Pi / 180 + θ1n),
   θ2 -> (4.123 * 1 * Pi / 180 + θ2n), θ3 -> (-4.123 * 1 * Pi / 180 + θ3n),
   θ4 -> (-60 * 0 * Pi / 180 + θ4n)} // MatrixForm
baseXFMframe4 /.
   {θ1 -> (-60 * 0 * Pi / 180 + θ1n), θ2 -> (4.123 * 1 * Pi / 180 + θ2n),
    θ3 -> (-4.123 * 1 * Pi / 180 + θ3n), θ4 -> (-60 * 0 * Pi / 180 + θ4n)} /.
  {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
   d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} // MatrixForm
aperXFMframe0 . baseXFMframe4 . a5 . a6 . frame6XFMhotspot /.
   {θ1 -> (-60 * 0 * Pi / 180 + θ1n), θ2 -> (4.123 * 1 * Pi / 180 + θ2n),
    θ3 -> (-4.123 * 1 * Pi / 180 + θ3n), θ4 -> (-60 * 0 * Pi / 180 + θ4n)} /.
  {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
   d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} // MatrixForm
% /. {θ5 -> θ5n - Pi / 2, θ6 -> θ6n} // MatrixForm
```

```
( -1   0    0        -aa4 + d2 + d3        )
(  0   0.   1.   0.997412 aa2 + 0. d4      )
(  0   1.   0.   0.0718978 aa2 - 1. d4     )
(  0   0    0              1               )
```

```
( -1   0    0      -21.4       )
(  0   0.   1.     62.438      )
(  0   1.   0.   0.000804593   )
(  0   0    0         1        )
```

```
( -1. Sin[θ5]        -1. Cos[θ5] Cos[θ6]                   -1. Cos[θ5] Sin[θ6]                  )
(  0. Cos[θ5]   0. Cos[θ6] Sin[θ5] + 1. Sin[θ6]   -1. Cos[θ6] + 0. Sin[θ5] Sin[θ6]   )
(  1. Cos[θ5]  -1. Cos[θ6] Sin[θ5] + 0. Sin[θ6]    0. Cos[θ6] - 1. Sin[θ5] Sin[θ6]   )
(      0.                 0. Sin[θ6]                          0. Cos[θ6]             )
```

```
( 1.   0    0   3.55271×10^-15 )
(  0   1.   0.     -105.238    )
(  0   0.   1.   0.000804593   )
( 0.   0.   0         1.       )
```

```
Simplify[solnframe4 /. theta4soln[[4]][[1]]] // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -aa4 + \frac{\sqrt{aa2^2-(d4+pz)^2}\ \left(-\sqrt{(d2+d3)^2\ (aa2^2-aa4^2+d2^2+2\ d2\ d3+d3^2-d4^2-2\ aa4\ px-px^2-2\ d4\ pz-pz^2)}\ +aa4\ \sqrt{}\right.}{aa2^2+d2^2+2\ d2\ d3+d3^2-d4^2-2\ d4\ pz-pz^2} \\ 0 & 0 & 1 & \frac{(d2+d3)\ \left(\sqrt{(d2+d3)^2\ (aa2^2-aa4^2+d2^2+2\ d2\ d3+d3^2-d4^2-2\ aa4\ px-px^2-2\ d4\ pz-pz^2)}\ -aa4\ \sqrt{aa2^2-(d4+pz)^2}\right.}{aa2^2+d2^2+2\ d2\ d3+d3^2-d4^2-2\ d4\ pz-pz^2} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[solnframe4 /. {theta4soln[[4]][[1]]} /.
    {px -> -aa4 + d2 + d3, pz -> 0}] /. {aa2 -> 62.6, aa4 -> 46,
   aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -70.6 \\ 0 & 0 & 1 & -62.438 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Simplify[solnframe4 /. theta4soln[[1]][[1]]] // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -aa4 + \frac{\sqrt{aa2^2-(d4+pz)^2}\ \left(\sqrt{(d2+d3)^2\ (aa2^2-aa4^2+d2^2+2\ d2\ d3+d3^2-d4^2-2\ aa4\ px-px^2-2\ d4\ pz-pz^2)}\ +aa4\ \sqrt{}\right.}{aa2^2+d2^2+2\ d2\ d3+d3^2-d4^2-2\ d4\ pz-pz^2} \\ 0 & 0 & 1 & -\frac{(d2+d3)\ \left(\sqrt{(d2+d3)^2\ (aa2^2-aa4^2+d2^2+2\ d2\ d3+d3^2-d4^2-2\ aa4\ px-px^2-2\ d4\ pz-pz^2)}\ +aa4\ \sqrt{aa2^2-(d4+pz)^2}\right.}{aa2^2+d2^2+2\ d2\ d3+d3^2-d4^2-2\ d4\ pz-pz^2} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[solnframe4 /. {theta4soln[[1]][[1]]} /.
    {px -> -aa4 + d2 + d3, pz -> 0}] /. {aa2 -> 62.6, aa4 -> 46,
   aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & 14.578 \\ 0 & 0 & 1 & 28.8787 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Simplify[solnframe4 /. theta4soln[[2]][[1]]] // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -aa4 + \frac{\sqrt{aa2^2-(d4+pz)^2}\left(\sqrt{(d2+d3)^2\left(aa2^2-aa4^2+d2^2+2\,d2\,d3+d3^2-d4^2-2\,aa4\,px-px^2-2\,d4\,pz-pz^2\right)}+aa4\sqrt{\ldots}\right.}{aa2^2+d2^2+2\,d2\,d3+d3^2-d4^2-2\,d4\,pz-pz^2} \\ 0 & 0 & 1 & -\frac{(d2+d3)\left(\sqrt{(d2+d3)^2\left(aa2^2-aa4^2+d2^2+2\,d2\,d3+d3^2-d4^2-2\,aa4\,px-px^2-2\,d4\,pz-pz^2\right)}+aa4\sqrt{aa2^2-(d4+pz)^2}\right.}{aa2^2+d2^2+2\,d2\,d3+d3^2-d4^2-2\,d4\,pz-pz^2} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[solnframe4 /. {theta4soln[[2]][[1]]} /.
    {px -> -aa4 + d2 + d3, pz -> 0}] /. {aa2 -> 62.6, aa4 -> 46,
   aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -21.4 \\ 0 & 0 & 1 & -62.438 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The following shows various placements of frame four origin relative to frame zero. Note that px and pz may be specified arbitrarily but py falls out as a result of this specification.

```
Simplify[solnframe4 /. {theta4soln[[3]][[1]]} /. {aa2 -> 62.6, aa4 -> 46,
     aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
   {px -> 21.1093883, pz -> 0}] //
 MatrixForm
```

```
-1  0  0    21.1094
 0  0  1  0.00141415
 0  1  0       0
 0  0  0       1
```

■ Positioning of Frame Four (less conservative approach)

Now assume only that frame four z-axis must align with frame zero y-axis (which is base frame -y-axis). Then see if frame four origin may be placed arbitrarily wrt frame zero.

```
baseXFMframe4 // MatrixForm
baseXFMframe4 /. {θ1 -> (-60 * 0 * Pi / 180 + θ1n),
   θ2 -> (4.123 * 1 * Pi / 180 + θ2n), θ3 -> (-4.123 * 1 * Pi / 180 + θ3n),
   θ4 -> (-60 * 0 * Pi / 180 + θ4n)} // MatrixForm
t4reqd = {{nx, ox, 0, px}, {ny, oy, 1, py}, {nz, oz, 0, pz}, {0, 0, 0, 1}};
t4reqd // MatrixForm
```

```
Cos[θ1] Cos[θ2 + θ3] Cos[θ4] + Sin[θ1] Sin[θ4]   -Cos[θ1] Sin[θ2 + θ3]   Cos[θ4]
Cos[θ2 + θ3] Cos[θ4] Sin[θ1] - Cos[θ1] Sin[θ4]   -Sin[θ1] Sin[θ2 + θ3]   -Cos[θ1]
            Cos[θ4] Sin[θ2 + θ3]                        Cos[θ2 + θ3]
                      0                                      0
```

```
-1   0   0        -aa4 + d2 + d3
 0   0.  1.   0.997412 aa2 + 0. d4
 0   1.  0.  0.0718978 aa2 - 1. d4
 0   0   0              1
```

```
nx  ox  0  px
ny  oy  1  py
nz  oz  0  pz
 0   0  0   1
```

From the 3,3 elements either $\theta 2+\theta 3$ = n Pi or $\theta 4$ = n Pi, where n is an integer. Start with $\theta 4$ = n Pi, then

```
baseXFMframe4 /. {Sin[θ4] -> 0} // MatrixForm
t4reqd // MatrixForm
```

```
Cos[θ1] Cos[θ2 + θ3] Cos[θ4]   -Cos[θ1] Sin[θ2 + θ3]   Cos[θ4] Sin[θ1]    (d2 + d3
Cos[θ2 + θ3] Cos[θ4] Sin[θ1]   -Sin[θ1] Sin[θ2 + θ3]   -Cos[θ1] Cos[θ4]  -(d2 + d3
    Cos[θ4] Sin[θ2 + θ3]           Cos[θ2 + θ3]                0
             0                          0                      0
```

```
nx  ox  0  px
ny  oy  1  py
nz  oz  0  pz
 0   0  0   1
```

The element 1,3 yields $\theta1$ = n Pi, where n is an integer, since Cos[$\theta4$] = +/-1, thus

```
baseXFMframe4 /. {Sin[θ4] -> 0} /. {Sin[θ1] -> 0} // MatrixForm
t4reqd // MatrixForm
```

```
Cos[θ1] Cos[θ2 + θ3] Cos[θ4]   -Cos[θ1] Sin[θ2 + θ3]           0            Cos[θ1]
             0                          0              -Cos[θ1] Cos[θ4]
    Cos[θ4] Sin[θ2 + θ3]           Cos[θ2 + θ3]                0               -d4
             0                          0                      0
```

```
nx  ox  0  px
ny  oy  1  py
nz  oz  0  pz
 0   0  0   1
```

Note at this point that control over specifying the position of frame four relative to frame zero is lost since Cos[$\theta1$] is already restricted to be +/-1!!! This value is fixed at +/-(-d2-d3).

Proceeding, element 2,3 shows nPi=$\theta1$=$\theta4$+Pi=(n+1)Pi, n integer, thus

```
baseXFMframe4 /. {Sin[θ4] -> 0} /. {Sin[θ1] -> 0} /.
  {θ4 -> θ1 - Pi} // MatrixForm
t4reqd // MatrixForm
```

```
( -Cos[θ1]^2 Cos[θ2 + θ3]   -Cos[θ1] Sin[θ2 + θ3]   0            Cos[θ1] (aa2 Cos[θ2] -
           0                        0              Cos[θ1]^2
  -Cos[θ1] Sin[θ2 + θ3]          Cos[θ2 + θ3]       0               -d4 Cos[θ2 + θ3] +
           0                        0               0
```

```
( nx  ox  0  px )
( ny  oy  1  py )
( nz  oz  0  pz )
(  0   0  0   1 )
```

Now

```
baseXFMframe4 /. {Sin[θ4] -> 0} /. {Sin[θ1] -> 0} /. {θ4 -> θ1 - Pi} /.
  {Cos[θ1]^2 -> 1} // MatrixForm
t4reqd // MatrixForm
```

```
(     -Cos[θ2 + θ3]        -Cos[θ1] Sin[θ2 + θ3]  0  Cos[θ1] (aa2 Cos[θ2] - aa4 Cos[
           0                      0               1                        - (d2 + d3
  -Cos[θ1] Sin[θ2 + θ3]       Cos[θ2 + θ3]        0      -d4 Cos[θ2 + θ3] + aa2 Sin[θ
           0                      0               0
```

```
( nx  ox  0  px )
( ny  oy  1  py )
( nz  oz  0  pz )
(  0   0  0   1 )
```

CONCLUSION: 3 DOF of translational control is lost!!!!!

Now back to second case

```
baseXFMframe4 // MatrixForm
baseXFMframe4 /. {θ1 -> (-60 * 0 * Pi / 180 + θ1 n),
   θ2 -> (4.123 * 1 * Pi / 180 + θ2 n), θ3 -> (-4.123 * 1 * Pi / 180 + θ3 n),
   θ4 -> (-60 * 0 * Pi / 180 + θ4 n)} // MatrixForm
t4reqd = {{nx, ox, 0, px}, {ny, oy, 1, py}, {nz, oz, 0, pz}, {0, 0, 0, 1}};
t4reqd // MatrixForm
```

```
Cos[θ1] Cos[θ2 + θ3] Cos[θ4] + Sin[θ1] Sin[θ4]   -Cos[θ1] Sin[θ2 + θ3]   Cos[θ4]
Cos[θ2 + θ3] Cos[θ4] Sin[θ1] - Cos[θ1] Sin[θ4]   -Sin[θ1] Sin[θ2 + θ3]   -Cos[θ1]
            Cos[θ4] Sin[θ2 + θ3]                       Cos[θ2 + θ3]
                      0                                     0
```

```
-1   0    0        -aa4 + d2 + d3
 0   0.   1.   0.997412 aa2 + 0. d4
 0   1.   0.   0.0718978 aa2 - 1. d4
 0   0    0              1
```

```
nx  ox  0  px
ny  oy  1  py
nz  oz  0  pz
 0   0  0   1
```

This time user θ2+θ3 = n Pi, where n is an integer, then

```
baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} // MatrixForm
t4reqd // MatrixForm
```

```
Cos[θ1] Cos[θ2 + θ3] Cos[θ4] + Sin[θ1] Sin[θ4]        0          Cos[θ4] Sin[θ1] -
Cos[θ2 + θ3] Cos[θ4] Sin[θ1] - Cos[θ1] Sin[θ4]        0         -Cos[θ1] Cos[θ4] -
                      0                          Cos[θ2 + θ3]
                      0                               0
```

```
nx  ox  0  px
ny  oy  1  py
nz  oz  0  pz
 0   0  0   1
```

It's known that Cos[θ2+θ3]=+/-1, try Cos[θ2+θ3]=+1, then

```
Simplify[
  baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /. {Cos[θ2 + θ3] -> 1}] // MatrixForm
t4reqd // MatrixForm
```

```
Cos[θ1 - θ4]  0   Sin[θ1 - θ4]   Cos[θ1] (aa2 Cos[θ2] + aa4 Cos[θ4]) + Sin[θ1] (d2
Sin[θ1 - θ4]  0  -Cos[θ1 - θ4]   (aa2 Cos[θ2] + aa4 Cos[θ4]) Sin[θ1] - Cos[θ1] (d2
     0        1        0                                 -d4 + aa2 Sin[θ2]
     0        0        0                                        1
```

```
nx  ox  0  px
ny  oy  1  py
nz  oz  0  pz
 0   0  0   1
```

Element 2,3 yields θ1=Pi+θ4, then

```
Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /. {Cos[θ2 + θ3] -> 1} /.
   {θ1 -> Pi + θ4}] // MatrixForm
t4reqd // MatrixForm
```

```
-1  0  0  -aa4 - aa2 Cos[θ2] Cos[θ4] - (d2 + d3) Sin[θ4]
 0  0  1      (d2 + d3) Cos[θ4] - aa2 Cos[θ2] Sin[θ4]
 0  1  0                  -d4 + aa2 Sin[θ2]
 0  0  0                         1
```

```
nx  ox  0  px
ny  oy  1  py
nz  oz  0  pz
 0   0  0   1
```

Conclusion: 3 DOF of translational control is lost!!!!! Specifying θ4 & θ2 fixes 3 translational positions.

Now try Cos[θ2+θ3]=-1 (since it's known that Cos[θ2+θ3]=+/-1), then

```
Simplify[
  baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /. {Cos[θ2 + θ3] -> -1}] // MatrixForm
t4reqd // MatrixForm
```

```
-Cos[θ1 + θ4]   0    Sin[θ1 + θ4]    Cos[θ1] (aa2 Cos[θ2] - aa4 Cos[θ4]) + Sin[θ1]
-Sin[θ1 + θ4]   0   -Cos[θ1 + θ4]    (aa2 Cos[θ2] - aa4 Cos[θ4]) Sin[θ1] - Cos[θ1]
      0        -1         0                                   d4 + aa2 Sin[θ2]
      0         0         0                                          1
```

```
nx  ox  0  px
ny  oy  1  py
nz  oz  0  pz
 0   0  0   1
```

Element 3,1 & 3,2 yields $\theta 1+\theta 4$=Pi or $\theta 1$=Pi-$\theta 4$, thus

```
Clear[θ1]
Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /. {Cos[θ2 + θ3] -> -1} /.
   {θ1 -> Pi - θ4}] // MatrixForm
t4reqd // MatrixForm
```

```
1   0  0  aa4 - aa2 Cos[θ2] Cos[θ4] + (d2 + d3) Sin[θ4]
0   0  1     (d2 + d3) Cos[θ4] + aa2 Cos[θ2] Sin[θ4]
0  -1  0                  d4 + aa2 Sin[θ2]
0   0  0                         1
```

```
nx  ox  0  px
ny  oy  1  py
nz  oz  0  pz
 0   0  0   1
```

Conclusion: 3 DOF of translational control is lost!!!!! Specifying $\theta 4$ & $\theta 2$ fixes 3 translational positions.

CONCLUSION: 3 DOF of translational control is lost even for less restrictive assumption case (ie only that frame four z-axis must align with frame zero y-axis (which is base frame -y-axis)!!!!!

Now solve for the 2 DOF of translational control.

```
temp = Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /.
    {Cos[θ2 + θ3] -> 1} /. {θ1 -> Pi + θ4}];
temp // MatrixForm
t4reqd // MatrixForm
theta2 = Solve[temp[[3, 4]] == t4reqd[[3, 4]], θ2][[1]][[1]]
```

$$\begin{pmatrix} -1 & 0 & 0 & -\text{aa4} - \text{aa2}\,\text{Cos}[\theta 2]\,\text{Cos}[\theta 4] - (\text{d2} + \text{d3})\,\text{Sin}[\theta 4] \\ 0 & 0 & 1 & (\text{d2} + \text{d3})\,\text{Cos}[\theta 4] - \text{aa2}\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 4] \\ 0 & 1 & 0 & -\text{d4} + \text{aa2}\,\text{Sin}[\theta 2] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} nx & ox & 0 & px \\ ny & oy & 1 & py \\ nz & oz & 0 & pz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found.

$$\theta 2 \to \text{ArcSin}\left[\frac{\text{d4} + \text{pz}}{\text{aa2}}\right]$$

```
temp = Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /.
     {Cos[θ2 + θ3] -> 1} /. {θ1 -> Pi + θ4} /. theta2];
temp // MatrixForm
theta4 = Solve[temp[[2, 4]] == t4reqd[[2, 4]], θ4]
θ4 * 180 / Pi /. theta4 /. {aa2 -> 62.6, aa4 -> 46,
   aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
  {py -> 62.4379913034758793, pz -> 0}
```

$$\begin{pmatrix} -1 & 0 & 0 & -aa4 - aa2\sqrt{\frac{aa2^2-(d4+pz)^2}{aa2^2}}\ \mathrm{Cos}[\theta4] - (d2+d3)\ \mathrm{Sin}[\theta4] \\ 0 & 0 & 1 & d2\ \mathrm{Cos}[\theta4] + d3\ \mathrm{Cos}[\theta4] - aa2\sqrt{1-\frac{(d4+pz)^2}{aa2^2}}\ \mathrm{Sin}[\theta4] \\ 0 & 1 & 0 & pz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found.

$$\{\{\theta4 \to -\mathrm{ArcCos}[(-(-2\,d2 - 2\,d3)\,py - 2\sqrt{aa2^2 - d4^2 - 2\,d4\,pz - pz^2}\sqrt{aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - py^2 - 2\,d4\,pz - pz^2}) / (2\,(aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - 2\,d4\,pz - pz^2))]\},$$

$$\{\theta4 \to \mathrm{ArcCos}[(-(-2\,d2 - 2\,d3)\,py - 2\sqrt{aa2^2 - d4^2 - 2\,d4\,pz - pz^2}\sqrt{aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - py^2 - 2\,d4\,pz - pz^2}) / (2\,(aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - 2\,d4\,pz - pz^2))]\},$$

$$\{\theta4 \to -\mathrm{ArcCos}[(-(-2\,d2 - 2\,d3)\,py + 2\sqrt{aa2^2 - d4^2 - 2\,d4\,pz - pz^2}\sqrt{aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - py^2 - 2\,d4\,pz - pz^2}) / (2\,(aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - 2\,d4\,pz - pz^2))]\},$$

$$\{\theta4 \to \mathrm{ArcCos}[(-(-2\,d2 - 2\,d3)\,py + 2\sqrt{aa2^2 - d4^2 - 2\,d4\,pz - pz^2}\sqrt{aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - py^2 - 2\,d4\,pz - pz^2}) / (2\,(aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - 2\,d4\,pz - pz^2))]\}\}$$

$$\{-90.0001, 90.0001, -46.9919, 46.9919\}$$

```
(* above says use results from
  theta[[1]][[1]] since it's closest to nominal configuration *)
theta4[[1]][[1]] /. {aa2 -> 62.6, aa4 -> 46,
   aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
 {py -> 62.4379913034758793, pz -> 0}
theta4[[1]][[1]]
```

$\theta 4 \to -1.5708$

$$\theta 4 \to -\text{ArcCos}\Big[\Big(-(-2\,d2 - 2\,d3)\,py - 2\sqrt{aa2^2 - d4^2 - 2\,d4\,pz - pz^2}\,\sqrt{aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - py^2 - 2\,d4\,pz - pz^2}\Big) \Big/ \left(2\left(aa2^2 + d2^2 + 2\,d2\,d3 + d3^2 - d4^2 - 2\,d4\,pz - pz^2\right)\right)\Big]$$

```
Clear[solution4]
solution4 =
 Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /. {Cos[θ2 + θ3] -> 1} /.
    {θ1 -> Pi + θ4} /. theta2 /. theta4[[1]][[1]]];
solution4 // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -\frac{\left(d2\,py + d3\,py - \sqrt{aa2^2+d2^2+2\,d2\,d3+d3^2-d4^2-py^2-2\,d4\,pz-pz^2}\,\sqrt{aa2^2-(d4+pz)^2}\right)\left(aa2\,(aa2^2+d2^2+2\,d2\,d3+d3^2 \cdots\right.}{(aa2^2+d2^2+2\,d2 \cdots} \\ 0 & 0 & 1 & \frac{\left(aa2\,(aa2^2+d2^2+2\,d2\,d3+d3^2-d4^2-2\,d4\,pz-pz^2)\sqrt{\frac{aa2^2-(d4+pz)^2}{aa2^2}} + aa4\,(d2\,py+d3\,py \cdots\right.}{\cdots} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
solution4 =
 Simplify[solution4 /. {Sqrt[(aa2^2 - (d4 + pz)^2)/aa2^2] -> Sqrt[aa2^2 - (d4 + pz)^2]/aa2}];
solution4 // MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -\frac{\left(d2\ py+d3\ py-\sqrt{aa2^2+d2^2+2\ d2\ d3+d3^2-d4^2-py^2-2\ d4\ pz-pz^2}\ \sqrt{aa2^2-(d4+pz)^2}\right)\left((aa2^2+d2^2+2\ d2\ d3+d3^2+d4 \ldots}{(aa2^2+d2^2+2\ d2 \ldots} \\ 0 & 0 & 1 & \frac{\left((aa2^2+d2^2+2\ d2\ d3+d3^2-d4^2-2\ d4\ pz-pz^2)\ \sqrt{aa2^2-(d4+pz)^2}+aa4\ (d2\ py+d3\ py \ldots}{} \\ 0 & 1 & 0 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
solution4 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
  {pz -> 0, py -> 62.4379913034758793} //
 MatrixForm
```

$$\begin{pmatrix} -1 & 0 & 0 & -21.3999 \\ 0 & 0 & 1 & 62.438 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now look at impact of this result on the hotspot frame wrt base frame.

```
Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /. {Cos[θ2 + θ3] -> 1} /.
   {θ1 -> Pi + θ4}] // MatrixForm
Simplify[baseXFMframe4 /. {θ3 -> -θ2} /. {θ1 -> Pi + θ4}] // MatrixForm
aperXFMframe0 . Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /.
      {Cos[θ2 + θ3] -> 1} /. {θ1 -> Pi + θ4}] .
   a5 . a6 . frame6XFMhotspot /.
  {0. -> 0, 1. -> 1, -1. -> -1} // MatrixForm
aperXFMframe0 . Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /.
      {Cos[θ2 + θ3] -> 1} /. {θ1 -> Pi + θ4}] .
   a5 . a6 . frame6XFMhotspot /.
  {0. -> 0, 1. -> 1, -1. -> -1} //
 MatrixForm
```

```
-1  0  0  -aa4 - aa2 Cos[θ2] Cos[θ4] - (d2 + d3) Sin[θ4]
 0  0  1     (d2 + d3) Cos[θ4] - aa2 Cos[θ2] Sin[θ4]
 0  1  0                -d4 + aa2 Sin[θ2]
 0  0  0                        1
```

```
-1  0  0  -aa4 - aa2 Cos[θ2] Cos[θ4] - (d2 + d3) Sin[θ4]
 0  0  1     (d2 + d3) Cos[θ4] - aa2 Cos[θ2] Sin[θ4]
 0  1  0                -d4 + aa2 Sin[θ2]
 0  0  0                        1
```

```
-Sin[θ5]   -Cos[θ5] Cos[θ6]   -Cos[θ5] Sin[θ6]   -21.4 + aa4 + aa2 Cos[θ2] Cos[θ4
   0            Sin[θ6]            -Cos[θ6]               -d5 - (d2 + d3) Cos[θ4] +
 Cos[θ5]   -Cos[θ6] Sin[θ5]   -Sin[θ5] Sin[θ6]                      -d4 + aa2 Sin[θ
   0              0                  0
```

```
-Sin[θ5]   -Cos[θ5] Cos[θ6]   -Cos[θ5] Sin[θ6]   -21.4 + aa4 + aa2 Cos[θ2] Cos[θ4
   0            Sin[θ6]            -Cos[θ6]               -d5 - (d2 + d3) Cos[θ4] +
 Cos[θ5]   -Cos[θ6] Sin[θ5]   -Sin[θ5] Sin[θ6]                      -d4 + aa2 Sin[θ
   0              0                  0
```

Given that the z-axis of frame four is aligned with the y-axis of the base frame, element 2,2 results in

```
aperXFMframe0 . Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /.
       {Cos[θ2 + θ3] -> 1} /. {θ1 -> Pi + θ4}] .
    a5 . a6 . frame6XFMhotspot /.
   {0. -> 0, 1. -> 1, -1. -> -1} /. {θ6 -> Pi / 2} //
 MatrixForm
```

$$\begin{pmatrix} -\text{Sin}[\theta 5] & 0 & -\text{Cos}[\theta 5] & -21.4 + \text{aa4} + \text{aa2}\,\text{Cos}[\theta 2]\,\text{Cos}[\theta 4] + (\text{d2} + \text{d3})\,\text{Sin}[\theta 4] \\ 0 & 1 & 0 & -\text{aa6} - \text{d5} - (\text{d2} + \text{d3})\,\text{Cos}[\theta 4] + \text{aa2}\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 4] \\ \text{Cos}[\theta 5] & 0 & -\text{Sin}[\theta 5] & -\text{d4} + \text{aa2}\,\text{Sin}[\theta 2] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Given that the z-axis of frame four is aligned with the y-axis of the base frame, elements 1,3 and 3,3 results in

```
aperXFMframe0 . Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /.
        {Cos[θ2 + θ3] -> 1} /. {θ1 -> Pi + θ4}] .
     a5 . a6 . frame6XFMhotspot /.
    {0. -> 0, 1. -> 1, -1. -> -1} /.
   {θ6 -> Pi / 2} /. {θ5 -> -Pi / 2} //
 MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & -21.4 + \text{aa4} + \text{aa2}\,\text{Cos}[\theta 2]\,\text{Cos}[\theta 4] + (\text{d2} + \text{d3})\,\text{Sin}[\theta 4] \\ 0 & 1 & 0 & -\text{aa6} - \text{d5} - (\text{d2} + \text{d3})\,\text{Cos}[\theta 4] + \text{aa2}\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 4] \\ 0 & 0 & 1 & -\text{d4} + \text{aa2}\,\text{Sin}[\theta 2] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

CONCLUSION: 3 DOF of translational control of the hotspot frame relative to the base frame isn't attainable (only 2 DOF are attainable) even for the less restrictive assumption case (ie only that frame four z-axis must align with frame zero y-axis (which is base frame -y-axis)!!!!! Numerically it becomes

```
aperXFMframe0 . Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /.
        {Cos[θ2 + θ3] -> 1} /. {θ1 -> Pi + θ4}] .
      a5 . a6 . frame6XFMhotspot /.
     {0. -> 0, 1. -> 1, -1. -> -1} /.
    {θ6 -> Pi / 2} /. {θ5 -> -Pi / 2} /.
  {θ4 -> θ4n, θ2 -> (4.123 * 1 * Pi / 180 + θ2n)} // MatrixForm
N[aperXFMframe0 . Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /.
         {Cos[θ2 + θ3] -> 1} /. {θ1 -> Pi + θ4}] .
       a5 . a6 . frame6XFMhotspot /.
      {0. -> 0, 1. -> 1, -1. -> -1} /.
     {θ6 -> Pi / 2} /. {θ5 -> -Pi / 2} /.
    {θ4 -> θ4n, θ2 -> (ArcSin[d4 / aa2] + θ2n)} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & -21.4 + aa4 - d2 - d3 \\ 0 & 1 & 0 & -0.997412\, aa2 - aa6 - d5 \\ 0 & 0 & 1 & 0.0718978\, aa2 - d4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0 & 0 & 3.55271 \times 10^{-15} \\ 0 & 1. & 0 & -105.238 \\ 0 & 0 & 1. & 0 \\ 0 & 0 & 0 & 1. \end{pmatrix}$$

```
Simplify[aperXFMframe0 . solution4 . a5 . a6 . frame6XFMhotspot /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {θ6 -> Pi / 2} /.
    {θ5 -> -Pi / 2} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & -21.4 + \frac{1. \left(24.6\, py - \sqrt{-1. (-58.1+pz)(67.1+pz)}\; \sqrt{-py^2 - 1. (-62.7601+pz)(71.7601+pz)}\right) \left(1131.6\, py + \sqrt{\cdots}\right.}{(-62.7601+pz)^2 (\cdots} \\ 0 & 1 & 0 & -42.8 + \frac{1. \left(\left(1131.6\, py + \sqrt{-1. (-58.1+pz)(67.1+pz)}\; \left(4503.67 - 9.\, pz - 1.\, pz^2 - \cdots\right.\right.\right.}{\cdots} \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[aperXFMframe0 . solution4 . a5 . a6 . frame6XFMhotspot /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {θ6 -> Pi / 2} /. {θ5 -> -Pi / 2} /.
    {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
     d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. {pz -> 0}] //
 MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 24.6 - 1.73472\times 10^{-18}\,py^2 - 0.86563\sqrt{4503.67 - py^2} + py\,(0.34105 - 8.67362 \\ 0 & 1 & 0 & -42.8 - 0.13437\,py + 0.34105\sqrt{4503.67 - py^2} - 62.438 \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \end{pmatrix}$$

```
Simplify[aperXFMframe0 . solution4 . a5 . a6 . frame6XFMhotspot /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {θ6 -> Pi / 2} /.
     {θ5 -> -Pi / 2} /. {aa2 -> 62.6, aa4 -> 46,
     aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
   {pz -> 0, py -> 62.4379913034758793}] //
 MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & -0.000147194 \\ 0 & 1 & 0 & -105.238 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Simplify[aperXFMframe0 . solution4 . a5 . a6 . frame6XFMhotspot /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {θ6 -> Pi / 2} /.
     {θ5 -> -Pi / 2} /. {aa2 -> 62.6, aa4 -> 46,
     aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
   {pz -> 0.1, py -> 62.438 + 0.34}] //
 MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0.900154 \\ 0 & 1 & 0 & -105.578 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Assumming that frame four z-axis must align with frame zero y-axis (which is base frame -y-axis) (restricts relations between $\theta 2+\theta 3=0$ & $\theta 1-\theta 4=$Pi/), the inverse kinematic relations are determined

```
Simplify[baseXFMframe4 /. {Sin[θ2 + θ3] -> 0} /. {Cos[θ2 + θ3] -> 1} /.
   {θ1 -> Pi + θ4}] // MatrixForm
Simplify[baseXFMframe4 /. {θ3 -> -θ2} /. {θ1 -> Pi + θ4}] // MatrixForm
Simplify[baseXFMframe4 /. {θ3 -> -θ2} /. {θ1 -> Pi + θ4}] . a5 . a6 /.
  {0. -> 0, 1. -> 1, -1. -> -1} // MatrixForm
(* aperXFMframe0.Simplify[baseXFMframe4/.{θ3->-θ2}/.{θ1->Pi+θ4}].
    a5.a6.frame6XFMhotspot/.
   {0.->0,1.->1,-1.->-1}//MatrixForm *)
t6reqd =
 {{nx, ox, ax, px}, {ny, oy, 0, py}, {nz, oz, az, pz}, {0, 0, 0, 1}};
MatrixForm[t6reqd]
```

```
( -1  0  0  -aa4 - aa2 Cos[θ2] Cos[θ4] - (d2 + d3) Sin[θ4] )
(  0  0  1     (d2 + d3) Cos[θ4] - aa2 Cos[θ2] Sin[θ4]     )
(  0  1  0              -d4 + aa2 Sin[θ2]                  )
(  0  0  0                      1                          )
```

```
( -1  0  0  -aa4 - aa2 Cos[θ2] Cos[θ4] - (d2 + d3) Sin[θ4] )
(  0  0  1     (d2 + d3) Cos[θ4] - aa2 Cos[θ2] Sin[θ4]     )
(  0  1  0              -d4 + aa2 Sin[θ2]                  )
(  0  0  0                      1                          )
```

```
( -Cos[θ5] Cos[θ6]    Cos[θ5] Sin[θ6]    -Sin[θ5]   -aa4 - aa2 Cos[θ2] Cos[θ4] - aa6
      Sin[θ6]             Cos[θ6]            0        d5 + (d2 + d3) Cos[θ4] - aa2
   Cos[θ6] Sin[θ5]   -Sin[θ5] Sin[θ6]   -Cos[θ5]                 -d4 + aa2 Sin[θ2]
         0                   0               0
```

```
( nx  ox  ax  px )
( ny  oy  0   py )
( nz  oz  az  pz )
( 0   0   0   1  )
```

```
inva6 = Inverse[a6];
Simplify[inva6] // MatrixForm
t6reqdinva6 = Simplify[t6reqd . inva6];
t6reqdinva6 // MatrixForm
a1a2a3a4a5 = Simplify[baseXFMframe4 /. {θ3 -> -θ2} /. {θ1 -> Pi + θ4}] . a5 /.
  {0. -> 0, 1. -> 1, -1. -> -1};
a1a2a3a4a5 // MatrixForm
```

$$\begin{pmatrix} \text{Cos}[\theta 6] & \text{Sin}[\theta 6] & 0 & -\text{aa6} \\ -\text{Sin}[\theta 6] & \text{Cos}[\theta 6] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \text{nx Cos}[\theta 6] - \text{ox Sin}[\theta 6] & \text{ox Cos}[\theta 6] + \text{nx Sin}[\theta 6] & \text{ax} & -\text{aa6 nx} + \text{px} \\ \text{ny Cos}[\theta 6] - \text{oy Sin}[\theta 6] & \text{oy Cos}[\theta 6] + \text{ny Sin}[\theta 6] & 0 & -\text{aa6 ny} + \text{py} \\ \text{nz Cos}[\theta 6] - \text{oz Sin}[\theta 6] & \text{oz Cos}[\theta 6] + \text{nz Sin}[\theta 6] & \text{az} & -\text{aa6 nz} + \text{pz} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -\text{Cos}[\theta 5] & 0 & -\text{Sin}[\theta 5] & -\text{aa4} - \text{aa2 Cos}[\theta 2]\,\text{Cos}[\theta 4] - (\text{d2} + \text{d3})\,\text{Sin}[\theta 4] \\ 0 & 1 & 0 & \text{d5} + (\text{d2} + \text{d3})\,\text{Cos}[\theta 4] - \text{aa2 Cos}[\theta 2]\,\text{Sin}[\theta 4] \\ \text{Sin}[\theta 5] & 0 & -\text{Cos}[\theta 5] & -\text{d4} + \text{aa2 Sin}[\theta 2] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Simplify[a1a2a3a4a5 /. {θ5 -> ArcTan[-az, -ax]}] // MatrixForm
```

$$\begin{pmatrix} -\text{Cos}[\text{ArcTan}[-\text{az}, -\text{ax}]] & 0 & -\text{Sin}[\text{ArcTan}[-\text{az}, -\text{ax}]] & -\text{aa4} - \text{aa2 Cos}[\theta 2]\,\text{Cos}[\theta 4 \\ 0 & 1 & 0 & \text{d5} + (\text{d2} + \text{d3})\,\text{Cos}[\theta 4] - \text{aa2} \\ \text{Sin}[\text{ArcTan}[-\text{az}, -\text{ax}]] & 0 & -\text{Cos}[\text{ArcTan}[-\text{az}, -\text{ax}]] & -\text{d4} + \text{aa2 Sin} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
t6reqd // MatrixForm
a1a2a3a4a5a6 =
 Simplify[baseXFMframe4 /. {θ3 -> -θ2} /. {θ1 -> Pi + θ4}] . a5 . a6 /.
   {0. -> 0, 1. -> 1, -1. -> -1} /. {θ5 -> ArcTan[-az, -ax]};
a1a2a3a4a5a6 // MatrixForm
```

```
( nx  ox  ax  px )
( ny  oy  0   py )
( nz  oz  az  pz )
( 0   0   0   1  )
```

```
( -Cos[θ6] Cos[ArcTan[-az, -ax]]   Cos[ArcTan[-az, -ax]] Sin[θ6]   -Sin[ArcTan
              Sin[θ6]                         Cos[θ6]
   Cos[θ6] Sin[ArcTan[-az, -ax]]  -Sin[θ6] Sin[ArcTan[-az, -ax]]  -Cos[ArcTan
                 0                               0
```

```
θ5 /. {θ5 -> ArcTan[-az, -ax]} /. {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0}
θ6 /. {θ5 -> ArcTan[-az, -ax]} /. {θ6 -> ArcTan[oy, ny]} /.
 {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0}
```

$-\frac{\pi}{2}$

$\frac{\pi}{2}$

```
t6reqd // MatrixForm
a1a2a3a4a5a6 = Simplify[
  Simplify[baseXFMframe4 /. {θ3 -> -θ2} /. {θ1 -> Pi + θ4}] .a5 .a6 /.
     {0. -> 0, 1. -> 1, -1. -> -1} /.
    {θ5 -> ArcTan[-az, -ax]} /. {θ6 -> ArcTan[oy, ny]}];
a1a2a3a4a5a6 // MatrixForm
a1a2a3a4a5a6 /. {ny -> +1, ox -> 1, oy -> 0, ax -> 0, az -> -1} /. {ay -> 0};
```

$$\begin{pmatrix} nx & ox & ax & px \\ ny & oy & 0 & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
( -Cos[ArcTan[-az, -ax]] Cos[ArcTan[oy, ny]]   Cos[ArcTan[-az, -ax]] Sin[ArcTan
              Sin[ArcTan[oy, ny]]                           Cos[ArcTan[oy, ny]]
  Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]   -Sin[ArcTan[-az, -ax]] Sin[ArcTan
                        0                                           0
```

```
theta2 = Solve[a1a2a3a4a5a6[[3, 4]] == t6reqd[[3, 4]], θ2][[1]][[1]]
temp = Simplify[a1a2a3a4a5a6 /. theta2];
temp // MatrixForm
```

Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found.

θ2 → ArcSin[(d4 + pz - aa6 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]) / aa2]

```
( -Cos[ArcTan[-az, -ax]] Cos[ArcTan[oy, ny]]   Cos[ArcTan[-az, -ax]] Sin[ArcTan

              Sin[ArcTan[oy, ny]]                           Cos[ArcTan[oy, ny]]
  Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]   -Sin[ArcTan[-az, -ax]] Sin[ArcTan
                        0                                           0
```

```
theta4 = Solve[temp[[2, 4]] == t6reqd[[2, 4]], θ4]
θ4 * 180 / Pi /. theta4 /. {aa2 -> 62.6, aa4 -> 46,
     aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
   {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
 {pz -> 0, py -> 105.237991303475886`}
```

Solve::ifun :
Inverse functions are being used by Solve, so some solutions may not be found.

```
{{θ4 → -ArcCos[
      (-2 d2 d5 - 2 d3 d5 + 2 d2 py + 2 d3 py - 2 aa6 d2 Sin[ArcTan[oy, ny]] -
        2 aa6 d3 Sin[ArcTan[oy, ny]] - 2 √(aa2^4 + aa2^2 d2^2 +
              2 aa2^2 d2 d3 + aa2^2 d3^2 - 2 aa2^2 d4^2 - d2^2 d4^2 - 2 d2 d3 d4^2 -
              d3^2 d4^2 + d4^4 - aa2^2 d5^2 + d4^2 d5^2 + 2 aa2^2 d5 py - 2 d4^2 d5 py -
              aa2^2 py^2 + d4^2 py^2 - 4 aa2^2 d4 pz - 2 d2^2 d4 pz - 4 d2 d3 d4 pz -
              2 d3^2 d4 pz + 4 d4^3 pz + 2 d4 d5^2 pz - 4 d4 d5 py pz +
              2 d4 py^2 pz - 2 aa2^2 pz^2 - d2^2 pz^2 - 2 d2 d3 pz^2 - d3^2 pz^2 +
              6 d4^2 pz^2 + d5^2 pz^2 - 2 d5 py pz^2 + py^2 pz^2 + 4 d4 pz^3 + pz^4 +
              4 aa2^2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
              2 aa6 d2^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
              4 aa6 d2 d3 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
              2 aa6 d3^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
              4 aa6 d4^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
              2 aa6 d4 d5^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
              4 aa6 d4 d5 py Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
              2 aa6 d4 py^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
              4 aa2^2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
              2 aa6 d2^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
              4 aa6 d2 d3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
              2 aa6 d3^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
              12 aa6 d4^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
              2 aa6 d5^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
              4 aa6 d5 py pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
              2 aa6 py^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
              12 aa6 d4 pz^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
              4 aa6 pz^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
              2 aa2^2 aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
              aa6^2 d2^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
              2 aa6^2 d2 d3 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
              aa6^2 d3^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
              6 aa6^2 d4^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
              aa6^2 d5^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
              2 aa6^2 d5 py Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
              aa6^2 py^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
              12 aa6^2 d4 pz Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
```

```
6 aa6^2 pz^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
4 aa6^3 d4 Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 -
4 aa6^3 pz Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 + aa6^4
 Cos[ArcTan[oy, ny]]^4 Sin[ArcTan[-az, -ax]]^4 - 2 aa2^2 aa6 d5
 Sin[ArcTan[oy, ny]] + 2 aa6 d4^2 d5 Sin[ArcTan[oy, ny]] +
2 aa2^2 aa6 py Sin[ArcTan[oy, ny]] - 2 aa6 d4^2 py
 Sin[ArcTan[oy, ny]] + 4 aa6 d4 d5 pz Sin[ArcTan[oy, ny]] -
4 aa6 d4 py pz Sin[ArcTan[oy, ny]] + 2 aa6 d5 pz^2
 Sin[ArcTan[oy, ny]] - 2 aa6 py pz^2 Sin[ArcTan[oy, ny]] -
4 aa6^2 d4 d5 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
 Sin[ArcTan[oy, ny]] + 4 aa6^2 d4 py Cos[ArcTan[oy, ny]]
 Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] -
4 aa6^2 d5 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
 Sin[ArcTan[oy, ny]] + 4 aa6^2 py pz Cos[ArcTan[oy, ny]]
 Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] +
2 aa6^3 d5 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2
 Sin[ArcTan[oy, ny]] - 2 aa6^3 py Cos[ArcTan[oy, ny]]^2
 Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]] -
aa2^2 aa6^2 Sin[ArcTan[oy, ny]]^2 + aa6^2 d4^2 Sin[ArcTan[oy, ny]]^2 +
2 aa6^2 d4 pz Sin[ArcTan[oy, ny]]^2 +
aa6^2 pz^2 Sin[ArcTan[oy, ny]]^2 - 2 aa6^3 d4 Cos[ArcTan[oy, ny]]
 Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]]^2 -
2 aa6^3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
 Sin[ArcTan[oy, ny]]^2 + aa6^4 Cos[ArcTan[oy, ny]]^2
 Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]]^2)) /
(2 (aa2^2 + d2^2 + 2 d2 d3 + d3^2 - d4^2 - 2 d4 pz - pz^2 +
   2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
   2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
   aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2))]},
{θ4 → ArcCos[(-2 d2 d5 - 2 d3 d5 + 2 d2 py + 2 d3 py -
   2 aa6 d2 Sin[ArcTan[oy, ny]] - 2 aa6 d3 Sin[ArcTan[oy, ny]] -
   2 √(aa2^4 + aa2^2 d2^2 + 2 aa2^2 d2 d3 + aa2^2 d3^2 - 2 aa2^2 d4^2 - d2^2 d4^2 -
      2 d2 d3 d4^2 - d3^2 d4^2 + d4^4 - aa2^2 d5^2 + d4^2 d5^2 + 2 aa2^2 d5 py -
      2 d4^2 d5 py - aa2^2 py^2 + d4^2 py^2 - 4 aa2^2 d4 pz - 2 d2^2 d4 pz -
      4 d2 d3 d4 pz - 2 d3^2 d4 pz + 4 d4^3 pz + 2 d4 d5^2 pz - 4 d4 d5 py pz +
      2 d4 py^2 pz - 2 aa2^2 pz^2 - d2^2 pz^2 - 2 d2 d3 pz^2 - d3^2 pz^2 +
      6 d4^2 pz^2 + d5^2 pz^2 - 2 d5 py pz^2 + py^2 pz^2 + 4 d4 pz^3 + pz^4 +
      4 aa2^2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
      2 aa6 d2^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
      4 aa6 d2 d3 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
      2 aa6 d3^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
      4 aa6 d4^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
      2 aa6 d4 d5^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
      4 aa6 d4 d5 py Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
      2 aa6 d4 py^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
      4 aa2^2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
```

```
2 aa6 d2^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
4 aa6 d2 d3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
2 aa6 d3^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
12 aa6 d4^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
2 aa6 d5^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
4 aa6 d5 py pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
2 aa6 py^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
12 aa6 d4 pz^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
4 aa6 pz^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
2 aa2^2 aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
aa6^2 d2^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
2 aa6^2 d2 d3 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
aa6^2 d3^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
6 aa6^2 d4^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
aa6^2 d5^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
2 aa6^2 d5 py Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
aa6^2 py^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
12 aa6^2 d4 pz Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
6 aa6^2 pz^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
4 aa6^3 d4 Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 -
4 aa6^3 pz Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 + aa6^4
 Cos[ArcTan[oy, ny]]^4 Sin[ArcTan[-az, -ax]]^4 - 2 aa2^2 aa6 d5
 Sin[ArcTan[oy, ny]] + 2 aa6 d4^2 d5 Sin[ArcTan[oy, ny]] +
2 aa2^2 aa6 py Sin[ArcTan[oy, ny]] - 2 aa6 d4^2 py
 Sin[ArcTan[oy, ny]] + 4 aa6 d4 d5 pz Sin[ArcTan[oy, ny]] -
4 aa6 d4 py pz Sin[ArcTan[oy, ny]] + 2 aa6 d5 pz^2
 Sin[ArcTan[oy, ny]] - 2 aa6 py pz^2 Sin[ArcTan[oy, ny]] -
4 aa6^2 d4 d5 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
 Sin[ArcTan[oy, ny]] + 4 aa6^2 d4 py Cos[ArcTan[oy, ny]]
 Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] -
4 aa6^2 d5 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
 Sin[ArcTan[oy, ny]] + 4 aa6^2 py pz Cos[ArcTan[oy, ny]]
 Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] +
2 aa6^3 d5 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2
 Sin[ArcTan[oy, ny]] - 2 aa6^3 py Cos[ArcTan[oy, ny]]^2
 Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]] -
aa2^2 aa6^2 Sin[ArcTan[oy, ny]]^2 + aa6^2 d4^2 Sin[ArcTan[oy, ny]]^2 +
2 aa6^2 d4 pz Sin[ArcTan[oy, ny]]^2 +
aa6^2 pz^2 Sin[ArcTan[oy, ny]]^2 - 2 aa6^3 d4 Cos[ArcTan[oy, ny]]
 Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]]^2 -
2 aa6^3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
 Sin[ArcTan[oy, ny]]^2 + aa6^4 Cos[ArcTan[oy, ny]]^2
 Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]]^2)) /
(2 (aa2^2 + d2^2 + 2 d2 d3 + d3^2 - d4^2 - 2 d4 pz - pz^2 +
   2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
   2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
```

```
      aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2))]},
{θ4 → -ArcCos[(-2 d2 d5 - 2 d3 d5 + 2 d2 py + 2 d3 py -
      2 aa6 d2 Sin[ArcTan[oy, ny]] - 2 aa6 d3 Sin[ArcTan[oy, ny]] +
      2 √(aa2^4 + aa2^2 d2^2 + 2 aa2^2 d2 d3 + aa2^2 d3^2 - 2 aa2^2 d4^2 - d2^2 d4^2 -
         2 d2 d3 d4^2 - d3^2 d4^2 + d4^4 - aa2^2 d5^2 + d4^2 d5^2 + 2 aa2^2 d5 py -
         2 d4^2 d5 py - aa2^2 py^2 + d4^2 py^2 - 4 aa2^2 d4 pz - 2 d2^2 d4 pz -
         4 d2 d3 d4 pz - 2 d3^2 d4 pz + 4 d4^3 pz + 2 d4 d5^2 pz - 4 d4 d5 py pz +
         2 d4 py^2 pz - 2 aa2^2 pz^2 - d2^2 pz^2 - 2 d2 d3 pz^2 - d3^2 pz^2 +
         6 d4^2 pz^2 + d5^2 pz^2 - 2 d5 py pz^2 + py^2 pz^2 + 4 d4 pz^3 + pz^4 +
         4 aa2^2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
         2 aa6 d2^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
         4 aa6 d2 d3 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
         2 aa6 d3^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
         4 aa6 d4^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
         2 aa6 d4 d5^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
         4 aa6 d4 d5 py Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
         2 aa6 d4 py^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
         4 aa2^2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
         2 aa6 d2^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
         4 aa6 d2 d3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
         2 aa6 d3^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
         12 aa6 d4^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
         2 aa6 d5^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
         4 aa6 d5 py pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
         2 aa6 py^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
         12 aa6 d4 pz^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
         4 aa6 pz^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
         2 aa2^2 aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
         aa6^2 d2^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
         2 aa6^2 d2 d3 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
         aa6^2 d3^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
         6 aa6^2 d4^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
         aa6^2 d5^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
         2 aa6^2 d5 py Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
         aa6^2 py^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
         12 aa6^2 d4 pz Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
         6 aa6^2 pz^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
         4 aa6^3 d4 Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 -
         4 aa6^3 pz Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 + aa6^4
          Cos[ArcTan[oy, ny]]^4 Sin[ArcTan[-az, -ax]]^4 - 2 aa2^2 aa6 d5
          Sin[ArcTan[oy, ny]] + 2 aa6 d4^2 d5 Sin[ArcTan[oy, ny]] +
         2 aa2^2 aa6 py Sin[ArcTan[oy, ny]] - 2 aa6 d4^2 py
          Sin[ArcTan[oy, ny]] + 4 aa6 d4 d5 pz Sin[ArcTan[oy, ny]] -
         4 aa6 d4 py pz Sin[ArcTan[oy, ny]] + 2 aa6 d5 pz^2
          Sin[ArcTan[oy, ny]] - 2 aa6 py pz^2 Sin[ArcTan[oy, ny]] -
         4 aa6^2 d4 d5 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
```

```
            Sin[ArcTan[oy, ny]] + 4 aa6^2 d4 py Cos[ArcTan[oy, ny]]
            Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] -
           4 aa6^2 d5 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
            Sin[ArcTan[oy, ny]] + 4 aa6^2 py pz Cos[ArcTan[oy, ny]]
            Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] +
           2 aa6^3 d5 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2
            Sin[ArcTan[oy, ny]] - 2 aa6^3 py Cos[ArcTan[oy, ny]]^2
            Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]] -
           aa2^2 aa6^2 Sin[ArcTan[oy, ny]]^2 + aa6^2 d4^2 Sin[ArcTan[oy, ny]]^2 +
           2 aa6^2 d4 pz Sin[ArcTan[oy, ny]]^2 +
           aa6^2 pz^2 Sin[ArcTan[oy, ny]]^2 - 2 aa6^3 d4 Cos[ArcTan[oy, ny]]
            Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]]^2 -
           2 aa6^3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
            Sin[ArcTan[oy, ny]]^2 + aa6^4 Cos[ArcTan[oy, ny]]^2
            Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]]^2)) /
      (2 (aa2^2 + d2^2 + 2 d2 d3 + d3^2 - d4^2 - 2 d4 pz - pz^2 +
          2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2))]},
 {θ4 → ArcCos[(-2 d2 d5 - 2 d3 d5 + 2 d2 py + 2 d3 py -
       2 aa6 d2 Sin[ArcTan[oy, ny]] - 2 aa6 d3 Sin[ArcTan[oy, ny]] +
       2 √(aa2^4 + aa2^2 d2^2 + 2 aa2^2 d2 d3 + aa2^2 d3^2 - 2 aa2^2 d4^2 - d2^2 d4^2 -
           2 d2 d3 d4^2 - d3^2 d4^2 + d4^4 - aa2^2 d5^2 + d4^2 d5^2 + 2 aa2^2 d5 py -
           2 d4^2 d5 py - aa2^2 py^2 + d4^2 py^2 - 4 aa2^2 d4 pz - 2 d2^2 d4 pz -
           4 d2 d3 d4 pz - 2 d3^2 d4 pz + 4 d4^3 pz + 2 d4 d5^2 pz - 4 d4 d5 py pz +
           2 d4 py^2 pz - 2 aa2^2 pz^2 - d2^2 pz^2 - 2 d2 d3 pz^2 - d3^2 pz^2 +
           6 d4^2 pz^2 + d5^2 pz^2 - 2 d5 py pz^2 + py^2 pz^2 + 4 d4 pz^3 + pz^4 +
           4 aa2^2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
           2 aa6 d2^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
           4 aa6 d2 d3 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
           2 aa6 d3^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
           4 aa6 d4^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
           2 aa6 d4 d5^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
           4 aa6 d4 d5 py Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
           2 aa6 d4 py^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
           4 aa2^2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
           2 aa6 d2^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
           4 aa6 d2 d3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
           2 aa6 d3^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
           12 aa6 d4^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
           2 aa6 d5^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
           4 aa6 d5 py pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
           2 aa6 py^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
           12 aa6 d4 pz^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
           4 aa6 pz^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
           2 aa2^2 aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
```

```
        aa6^2 d2^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
        2 aa6^2 d2 d3 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
        aa6^2 d3^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
        6 aa6^2 d4^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
        aa6^2 d5^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
        2 aa6^2 d5 py Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
        aa6^2 py^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
        12 aa6^2 d4 pz Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
        6 aa6^2 pz^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
        4 aa6^3 d4 Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 -
        4 aa6^3 pz Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 + aa6^4
         Cos[ArcTan[oy, ny]]^4 Sin[ArcTan[-az, -ax]]^4 - 2 aa2^2 aa6 d5
         Sin[ArcTan[oy, ny]] + 2 aa6 d4^2 d5 Sin[ArcTan[oy, ny]] +
        2 aa2^2 aa6 py Sin[ArcTan[oy, ny]] - 2 aa6 d4^2 py
         Sin[ArcTan[oy, ny]] + 4 aa6 d4 d5 pz Sin[ArcTan[oy, ny]] -
        4 aa6 d4 py pz Sin[ArcTan[oy, ny]] + 2 aa6 d5 pz^2
         Sin[ArcTan[oy, ny]] - 2 aa6 py pz^2 Sin[ArcTan[oy, ny]] -
        4 aa6^2 d4 d5 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
         Sin[ArcTan[oy, ny]] + 4 aa6^2 d4 py Cos[ArcTan[oy, ny]]
         Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] -
        4 aa6^2 d5 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
         Sin[ArcTan[oy, ny]] + 4 aa6^2 py pz Cos[ArcTan[oy, ny]]
         Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] +
        2 aa6^3 d5 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2
         Sin[ArcTan[oy, ny]] - 2 aa6^3 py Cos[ArcTan[oy, ny]]^2
         Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]] -
        aa2^2 aa6^2 Sin[ArcTan[oy, ny]]^2 + aa6^2 d4^2 Sin[ArcTan[oy, ny]]^2 +
        2 aa6^2 d4 pz Sin[ArcTan[oy, ny]]^2 +
        aa6^2 pz^2 Sin[ArcTan[oy, ny]]^2 - 2 aa6^3 d4 Cos[ArcTan[oy, ny]]
         Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]]^2 -
        2 aa6^3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
         Sin[ArcTan[oy, ny]]^2 + aa6^4 Cos[ArcTan[oy, ny]]^2
         Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]]^2)) /
   (2 (aa2^2 + d2^2 + 2 d2 d3 + d3^2 - d4^2 - 2 d4 pz - pz^2 +
      2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
      2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
      aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2))]}}
```

```
{-90.0001, 90.0001, -46.9919, 46.9919}
```

Now assume manipulator is close to nominal orientation (to define $\theta 4$),

```
(* above says use results from
  theta[[1]][[1]] since it's closest to nominal configuration *)
theta4[[1]][[1]]
θ4 * 180 / Pi /. theta4[[1]][[1]] /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
   {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
 {pz -> 0, py -> 105.237991303475886`}
```

```
θ4 →
 -ArcCos[ (-2 d2 d5 - 2 d3 d5 + 2 d2 py + 2 d3 py - 2 aa6 d2 Sin[ArcTan[oy, ny]] -
      2 aa6 d3 Sin[ArcTan[oy, ny]] - 2 √(aa2^4 + aa2^2 d2^2 +
          2 aa2^2 d2 d3 + aa2^2 d3^2 - 2 aa2^2 d4^2 - d2^2 d4^2 - 2 d2 d3 d4^2 -
          d3^2 d4^2 + d4^4 - aa2^2 d5^2 + d4^2 d5^2 + 2 aa2^2 d5 py - 2 d4^2 d5 py -
          aa2^2 py^2 + d4^2 py^2 - 4 aa2^2 d4 pz - 2 d2^2 d4 pz -
          4 d2 d3 d4 pz - 2 d3^2 d4 pz + 4 d4^3 pz + 2 d4 d5^2 pz - 4 d4 d5 py pz +
          2 d4 py^2 pz - 2 aa2^2 pz^2 - d2^2 pz^2 - 2 d2 d3 pz^2 - d3^2 pz^2 +
          6 d4^2 pz^2 + d5^2 pz^2 - 2 d5 py pz^2 + py^2 pz^2 + 4 d4 pz^3 + pz^4 +
          4 aa2^2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          2 aa6 d2^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          4 aa6 d2 d3 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          2 aa6 d3^2 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          4 aa6 d4^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          2 aa6 d4 d5^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          4 aa6 d4 d5 py Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          2 aa6 d4 py^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          4 aa2^2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          2 aa6 d2^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          4 aa6 d2 d3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          2 aa6 d3^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          12 aa6 d4^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          2 aa6 d5^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
          4 aa6 d5 py pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          2 aa6 py^2 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          12 aa6 d4 pz^2 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          4 aa6 pz^3 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
          2 aa2^2 aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
          aa6^2 d2^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
          2 aa6^2 d2 d3 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
          aa6^2 d3^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
          6 aa6^2 d4^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
          aa6^2 d5^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
          2 aa6^2 d5 py Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
          aa6^2 py^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
          12 aa6^2 d4 pz Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 +
          6 aa6^2 pz^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2 -
```

```
4 aa6^3 d4 Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 -
4 aa6^3 pz Cos[ArcTan[oy, ny]]^3 Sin[ArcTan[-az, -ax]]^3 +
aa6^4 Cos[ArcTan[oy, ny]]^4 Sin[ArcTan[-az, -ax]]^4 - 2 aa2^2 aa6 d5
 Sin[ArcTan[oy, ny]] + 2 aa6 d4^2 d5 Sin[ArcTan[oy, ny]] +
2 aa2^2 aa6 py Sin[ArcTan[oy, ny]] - 2 aa6 d4^2 py
 Sin[ArcTan[oy, ny]] + 4 aa6 d4 d5 pz Sin[ArcTan[oy, ny]] -
4 aa6 d4 py pz Sin[ArcTan[oy, ny]] + 2 aa6 d5 pz^2
 Sin[ArcTan[oy, ny]] - 2 aa6 py pz^2 Sin[ArcTan[oy, ny]] -
4 aa6^2 d4 d5 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
 Sin[ArcTan[oy, ny]] + 4 aa6^2 d4 py Cos[ArcTan[oy, ny]]
 Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] -
4 aa6^2 d5 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
 Sin[ArcTan[oy, ny]] + 4 aa6^2 py pz Cos[ArcTan[oy, ny]]
 Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]] +
2 aa6^3 d5 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2
 Sin[ArcTan[oy, ny]] - 2 aa6^3 py Cos[ArcTan[oy, ny]]^2
 Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]] -
aa2^2 aa6^2 Sin[ArcTan[oy, ny]]^2 + aa6^2 d4^2 Sin[ArcTan[oy, ny]]^2 +
2 aa6^2 d4 pz Sin[ArcTan[oy, ny]]^2 +
aa6^2 pz^2 Sin[ArcTan[oy, ny]]^2 - 2 aa6^3 d4 Cos[ArcTan[oy, ny]]
 Sin[ArcTan[-az, -ax]] Sin[ArcTan[oy, ny]]^2 -
2 aa6^3 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]]
 Sin[ArcTan[oy, ny]]^2 + aa6^4 Cos[ArcTan[oy, ny]]^2
 Sin[ArcTan[-az, -ax]]^2 Sin[ArcTan[oy, ny]]^2)) /
(2 (aa2^2 + d2^2 + 2 d2 d3 + d3^2 - d4^2 - 2 d4 pz -
  pz^2 + 2 aa6 d4 Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] +
  2 aa6 pz Cos[ArcTan[oy, ny]] Sin[ArcTan[-az, -ax]] -
  aa6^2 Cos[ArcTan[oy, ny]]^2 Sin[ArcTan[-az, -ax]]^2))]
```

```
-90.0001
```

theta2

$$\theta 2 \to \text{ArcSin}\left[\frac{d4 + pz - aa6\,\text{Cos}[\text{ArcTan}[oy, ny]]\,\text{Sin}[\text{ArcTan}[-az, -ax]]}{aa2}\right]$$

The following specifications are frame6 wrt frame0 not hotspot frame to base frame.

```
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}

4.12226
```

```
tempθ3 = θ3 /. θ3 -> -%

-4.12226
```

```
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}

-90.0001
```

```
tempθ1 = θ1 /. {θ1 -> 180 + θ4} /. {θ4 -> %}

89.9999
```

```
tempθ5 =
 θ5 * 180 / Pi /. {θ5 -> ArcTan[-az, -ax]} /. {0. -> 0, 1. -> 1, -1. -> -1} /.
     {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
      d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}

-90
```

```
tempθ6 =
 θ6 * 180 / Pi /. {θ6 -> ArcTan[oy, ny]} /. {0. -> 0, 1. -> 1, -1. -> -1} /.
     {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
      d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}
```

```
90
```

```
tempθ1
tempθ2
tempθ3
tempθ4
tempθ5
tempθ6
```

```
89.9999
```

```
4.12226
```

```
-4.12226
```

```
-90.0001
```

```
-90
```

```
90
```

```
aperXFMframe0 . t6 . frame6XFMhotspot /.
    {θ1 -> (tempθ1 * Pi / 180), θ2 -> (tempθ2 * Pi / 180),
     θ3 -> (tempθ3 * Pi / 180), θ4 -> (tempθ4 * Pi / 180),
     θ5 -> (tempθ5 * Pi / 180), θ6 -> (tempθ6 * Pi / 180)} /.
   {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
   aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} //
 MatrixForm
```

$$\begin{pmatrix} 1 & 1.22461\times10^{-16} & 0 & -0.000147194 \\ -1.22461\times10^{-16} & 1 & 0 & -105.238 \\ 0 & 0 & 1 & 0. \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Simplify[a1a2a3a4a5a6 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] /.
   {ny -> 1, ox -> 1, oy -> 0, ax -> 0, az -> -1} /. {ay -> 0} // MatrixForm
Simplify[aperXFMframe0 . a1a2a3a4a5a6 . frame6XFMhotspot /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] /.
   {ny -> +1, ox -> 1, oy -> 0, ax -> 0, az -> -1} /. {ay -> 0} //
 MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 0 & -46 - 62.6\,\mathrm{Cos}[\theta 2]\,\mathrm{Cos}[\theta 4] - 24.6\,\mathrm{Sin}[\theta 4] \\ 1 & 0 & 0 & 42.8 + 24.6\,\mathrm{Cos}[\theta 4] - 62.6\,\mathrm{Cos}[\theta 2]\,\mathrm{Sin}[\theta 4] \\ 0 & 0 & -1 & -4.5 + 62.6\,\mathrm{Sin}[\theta 2] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & -1 & 62.6\,\mathrm{Cos}[\theta 2]\,\mathrm{Cos}[\theta 4] + 24.6\,(1 + \mathrm{Sin}[\theta 4]) \\ 0 & 1 & 0 & -42.8 - 24.6\,\mathrm{Cos}[\theta 4] + 62.6\,\mathrm{Cos}[\theta 2]\,\mathrm{Sin}[\theta 4] \\ 1 & 0 & 0 & -4.5 + 62.6\,\mathrm{Sin}[\theta 2] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}
```

4.12226

```
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}

-90.0001
```

```
Simplify[a1a2a3a4a5a6 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] /.
   {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} //
 MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 1 & -46 - 62.6\,\text{Cos}[\theta 2]\,\text{Cos}[\theta 4] - 24.6\,\text{Sin}[\theta 4] \\ 1 & 0 & 0 & 42.8 + 24.6\,\text{Cos}[\theta 4] - 62.6\,\text{Cos}[\theta 2]\,\text{Sin}[\theta 4] \\ 0 & 1 & 0 & -4.5 + 62.6\,\text{Sin}[\theta 2] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Simplify[aperXFMframe0 . % . frame6XFMhotspot /.
    θ2 -> Pi * tempθ2 / 180 /. θ4 -> Pi * tempθ4 / 180] //
 MatrixForm
```

$$\begin{pmatrix} 1. & 0. & 0. & -0.000147194 \\ 0. & 1. & 0. & -105.238 \\ 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

```
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}
Simplify[a1a2a3a4a5a6 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] /.
   {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} // MatrixForm
Simplify[aperXFMframe0 . % . frame6XFMhotspot /.
    θ2 -> Pi * tempθ2 / 180 /. θ4 -> Pi * tempθ4 / 180] //
 MatrixForm
```

```
4.12226
```

```
-90.0001
```

$$\begin{pmatrix} 0 & 0 & 1 & -46 - 62.6 \text{Cos}[\theta 2] \text{Cos}[\theta 4] - 24.6 \text{Sin}[\theta 4] \\ 1 & 0 & 0 & 42.8 + 24.6 \text{Cos}[\theta 4] - 62.6 \text{Cos}[\theta 2] \text{Sin}[\theta 4] \\ 0 & 1 & 0 & -4.5 + 62.6 \text{Sin}[\theta 2] \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & -0.000147194 \\ 0. & 1. & 0. & -105.238 \\ 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

The following specifications are hot spot frame wrt base frame (aperXFMhotspot). Note that position vector components are wrt frame four origin wrt base frame.

```
t6reqd // MatrixForm
aperXFMhotspot = Simplify[aperXFMframe0 . t6reqd . frame6XFMhotspot] /.
  {0. -> 0, 1. -> 1, -1. -> -1};
aperXFMhotspot // MatrixForm
```

$$\begin{pmatrix} nx & ox & ax & px \\ ny & oy & 0 & py \\ nz & oz & az & pz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

General::spell1 : Possible spelling error: new symbol name "aperXFMhotspot" is similar to existing symbol "taperXFMhotspot".

$$\begin{pmatrix} ax & nx & -ox & -21.4-px \\ 0 & ny & -oy & -py \\ -az & -nz & oz & pz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}
```

4.12226

```
tempθ3 = θ3 /. θ3 -> -%
```

-4.12226

```
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}
```

-90.0001

```
tempθ1 = θ1 /. {θ1 -> 180 + θ4} /. {θ4 -> %}

89.9999
```

```
tempθ5 =
 θ5 * 180 / Pi /. {θ5 -> ArcTan[-az, -ax]} /. {0. -> 0, 1. -> 1, -1. -> -1} /.
     {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
      d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}

-90
```

```
tempθ6 =
 θ6 * 180 / Pi /. {θ6 -> ArcTan[oy, ny]} /. {0. -> 0, 1. -> 1, -1. -> -1} /.
     {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
      d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}

90
```

```
tempθ1
tempθ2
tempθ3
tempθ4
tempθ5
tempθ6
```

```
89.9999
```

```
4.12226
```

```
-4.12226
```

```
-90.0001
```

```
-90
```

```
90
```

```
Chop[aperXFMframe0 . t6 . frame6XFMhotspot /.
     {θ1 -> (tempθ1 * Pi / 180), θ2 -> (tempθ2 * Pi / 180),
      θ3 -> (tempθ3 * Pi / 180), θ4 -> (tempθ4 * Pi / 180),
      θ5 -> (tempθ5 * Pi / 180), θ6 -> (tempθ6 * Pi / 180)} /.
    {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & -0.000147194 \\ 0 & 1 & 0 & -105.238 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}
tempθ3 = θ3 /. θ3 -> -%
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}
tempθ1 = θ1 /. {θ1 -> 180 + θ4} /. {θ4 -> %}
tempθ5 = θ5 * 180 / Pi /. {θ5 -> ArcTan[-az, -ax]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}
tempθ6 = θ6 * 180 / Pi /. {θ6 -> ArcTan[oy, ny]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886`}

tempθ1
tempθ2
tempθ3
tempθ4
tempθ5
tempθ6

Chop[aperXFMframe0 . t6 . frame6XFMhotspot /.
     {θ1 -> (tempθ1 * Pi / 180), θ2 -> (tempθ2 * Pi / 180),
      θ3 -> (tempθ3 * Pi / 180), θ4 -> (tempθ4 * Pi / 180),
      θ5 -> (tempθ5 * Pi / 180), θ6 -> (tempθ6 * Pi / 180)} /.
    {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

```
4.12226
```

```
-4.12226
```

```
-90.0001
```

```
89.9999
```

```
-90
```

```
90
```

```
89.9999
```

```
4.12226
```

```
-4.12226
```

```
-90.0001
```

```
-90
```

```
90
```

$$\begin{pmatrix} 1 & 0 & 0 & -0.000147194 \\ 0 & 1 & 0 & -105.238 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886 - 1}
tempθ3 = θ3 /. θ3 -> -%
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886 - 1}
tempθ1 = θ1 /. {θ1 -> 180 + θ4} /. {θ4 -> %}
tempθ5 = θ5 * 180 / Pi /. {θ5 -> ArcTan[-az, -ax]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886 - 1}
tempθ6 = θ6 * 180 / Pi /. {θ6 -> ArcTan[oy, ny]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886 - 1}

tempθ1
tempθ2
tempθ3
tempθ4
tempθ5
tempθ6

Chop[aperXFMframe0 . t6 . frame6XFMhotspot /.
     {θ1 -> (tempθ1 * Pi / 180), θ2 -> (tempθ2 * Pi / 180),
      θ3 -> (tempθ3 * Pi / 180), θ4 -> (tempθ4 * Pi / 180),
      θ5 -> (tempθ5 * Pi / 180), θ6 -> (tempθ6 * Pi / 180)} /.
    {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

```
4.12226
```

```
-4.12226
```

```
-92.2206
```

```
87.7794
```

```
-90
```

```
90
```

```
87.7794
```

```
4.12226
```

```
-4.12226
```

```
-92.2206
```

```
-90
```

```
90
```

$$\begin{pmatrix} 1 & 0 & 0 & -2.4008 \\ 0 & 1 & 0 & -104.238 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886 + 1}
tempθ3 = θ3 /. θ3 -> -%
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886 + 1}
tempθ1 = θ1 /. {θ1 -> 180 + θ4} /. {θ4 -> %}
tempθ5 = θ5 * 180 / Pi /. {θ5 -> ArcTan[-az, -ax]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886 + 1}
tempθ6 = θ6 * 180 / Pi /. {θ6 -> ArcTan[oy, ny]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 0, py -> 105.237991303475886 + 1}

tempθ1
tempθ2
tempθ3
tempθ4
tempθ5
tempθ6

Chop[aperXFMframe0 . t6 . frame6XFMhotspot /.
     {θ1 -> (tempθ1 * Pi / 180), θ2 -> (tempθ2 * Pi / 180),
      θ3 -> (tempθ3 * Pi / 180), θ4 -> (tempθ4 * Pi / 180),
      θ5 -> (tempθ5 * Pi / 180), θ6 -> (tempθ6 * Pi / 180)} /.
    {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

```
4.12226
```

```
-4.12226
```

```
-87.5358
```

```
92.4642
```

```
-90
```

```
90
```

```
92.4642
```

```
4.12226
```

```
-4.12226
```

```
-87.5358
```

```
-90
```

```
90
```

$$\begin{pmatrix} 1 & 0 & 0 & 2.70728 \\ 0 & 1 & 0 & -106.238 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 1, py -> 105.237991303475886}
tempθ3 = θ3 /. θ3 -> -%
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 1, py -> 105.237991303475886}
tempθ1 = θ1 /. {θ1 -> 180 + θ4} /. {θ4 -> %}
tempθ5 = θ5 * 180 / Pi /. {θ5 -> ArcTan[-az, -ax]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 1, py -> 105.237991303475886}
tempθ6 = θ6 * 180 / Pi /. {θ6 -> ArcTan[oy, ny]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> 1, py -> 105.237991303475886}

tempθ1
tempθ2
tempθ3
tempθ4
tempθ5
tempθ6

Chop[aperXFMframe0 . t6 . frame6XFMhotspot /.
     {θ1 -> (tempθ1 * Pi / 180), θ2 -> (tempθ2 * Pi / 180),
      θ3 -> (tempθ3 * Pi / 180), θ4 -> (tempθ4 * Pi / 180),
      θ5 -> (tempθ5 * Pi / 180), θ6 -> (tempθ6 * Pi / 180)} /.
    {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

```
5.04047
```

```
-5.04047
```

```
-89.8127
```

```
90.1873
```

```
-90
```

```
90
```

```
90.1873
```

```
5.04047
```

```
-5.04047
```

```
-89.8127
```

```
-90
```

```
90
```

$$\begin{pmatrix} 1 & 0 & 0 & 0.20395 \\ 0 & 1 & 0 & -105.238 \\ 0 & 0 & 1 & 1. \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6,
      aa4 -> 46, aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> -1, py -> 105.237991303475886}
tempθ3 = θ3 /. θ3 -> -%
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> -1, py -> 105.237991303475886}
tempθ1 = θ1 /. {θ1 -> 180 + θ4} /. {θ4 -> %}
tempθ5 = θ5 * 180 / Pi /. {θ5 -> ArcTan[-az, -ax]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> -1, py -> 105.237991303475886}
tempθ6 = θ6 * 180 / Pi /. {θ6 -> ArcTan[oy, ny]} /.
      {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
      aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /.
    {ny -> 1, ox -> 0, oy -> 0, ax -> 1, az -> 0} /. {ay -> 0} /.
  {pz -> -1, py -> 105.237991303475886}

tempθ1
tempθ2
tempθ3
tempθ4
tempθ5
tempθ6

Chop[aperXFMframe0 . t6 . frame6XFMhotspot /.
     {θ1 -> (tempθ1 * Pi / 180), θ2 -> (tempθ2 * Pi / 180),
      θ3 -> (tempθ3 * Pi / 180), θ4 -> (tempθ4 * Pi / 180),
      θ5 -> (tempθ5 * Pi / 180), θ6 -> (tempθ6 * Pi / 180)} /.
    {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

```
3.20511
```

```
-3.20511
```

```
-90.1488
```

```
89.8512
```

```
-90
```

```
90
```

```
89.8512
```

```
3.20511
```

```
-3.20511
```

```
-90.1488
```

```
-90
```

```
90
```

$$\begin{pmatrix} 1 & 0 & 0 & -0.162214 \\ 0 & 1 & 0 & -105.238 \\ 0 & 0 & 1 & -1. \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

aperXFMhotspot // MatrixForm

$$\begin{pmatrix} ax & nx & -ox & -21.4 - px \\ 0 & ny & -oy & -py \\ -az & -nz & oz & pz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

theta2

$$\theta 2 \to \text{ArcSin}\left[\frac{d4 + pz - aa6\ \text{Cos}[\text{ArcTan}[oy, ny]]\ \text{Sin}[\text{ArcTan}[-az, -ax]]}{aa2}\right]$$

```
finishang = {ny -> 1, ox -> 1, oy -> 0, ax -> 0, az -> -1};
finishposn = {pz -> 0, py -> 105.237991303475886};
tempθ2 = θ2 * 180 / Pi /. theta2 /. {0. -> 0, 1. -> 1, -1. -> -1} /.
      {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
       d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. finishang /.
   {ay -> 0} /. finishposn
tempθ3 = θ3 /. θ3 -> -%
tempθ4 = θ4 * 180 / Pi /. theta4[[1]][[1]] /. {0. -> 0, 1. -> 1, -1. -> -1} /.
      {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
       d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. finishang /.
   {ay -> 0} /. finishposn
tempθ1 = θ1 /. {θ1 -> 180 + θ4} /. {θ4 -> %}
tempθ5 =
 θ5 * 180 / Pi /. {θ5 -> ArcTan[-az, -ax]} /. {0. -> 0, 1. -> 1, -1. -> -1} /.
      {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
       d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. finishang /.
   {ay -> 0} /. finishposn
tempθ6 =
 θ6 * 180 / Pi /. {θ6 -> ArcTan[oy, ny]} /. {0. -> 0, 1. -> 1, -1. -> -1} /.
      {aa2 -> 62.6, aa4 -> 46, aa6 -> 3.1,
       d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7} /. finishang /.
   {ay -> 0} /. finishposn

tempθ1
tempθ2
tempθ3
tempθ4
tempθ5
tempθ6

Chop[aperXFMframe0 . t6 . frame6XFMhotspot /.
      {θ1 -> (tempθ1 * Pi / 180), θ2 -> (tempθ2 * Pi / 180),
       θ3 -> (tempθ3 * Pi / 180), θ4 -> (tempθ4 * Pi / 180),
       θ5 -> (tempθ5 * Pi / 180), θ6 -> (tempθ6 * Pi / 180)} /.
    {0. -> 0, 1. -> 1, -1. -> -1} /. {aa2 -> 62.6, aa4 -> 46,
    aa6 -> 3.1, d2 -> 6.9, d3 -> 17.7, d4 -> 4.5, d5 -> 39.7}] //
 MatrixForm
```

```
4.12226
```

```
-4.12226
```

```
-90.0001
```

```
89.9999
```

```
0
```

```
90
```

```
89.9999
```

```
4.12226
```

```
-4.12226
```

```
-90.0001
```

```
0
```

```
90
```

$$\begin{pmatrix} 0 & 0 & -1 & -0.000147194 \\ 0 & 1 & 0 & -105.238 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Equation of Motion

- **Potential Energy**
- **Kinematic Energy**
- **The Lagrangian**

Control Strategies

- **Disturbance Environment**
- **Local Loop Closure for Manipulator Compliance**
- **Coordinated Joint Control for Concentrator Pointing & Focus**

Performance Predictions

- **Frequency Domain**
- **Time Domain**

References