

***Oral Comprehensive Examination and
Ph.D. Dissertation Proposal Defense***

Presenter:

Delano R. Carter

Date:

May 29, 1996

Committee:

Dr. Armando A. Rodriguez, Chair

Dr. Walter T. Higgins

Dr. John N. McDonald

Dr. Jennie Si

Dr. Konstantinos Tsakalis

Outline

- ***Introduction and Motivation***
- ***Literature Survey***
- ***Problem Formulation***
- ***Solution Approach***
- ***Summary & Future Directions***

Introduction & Motivation

- ***Proposed Thesis Topic:***
 - H -Optimal Control of Distributed Parameter Sampled-Data Systems***
- ***Targeted Application Areas:***
 - **Industrial Control Processes many of which are formulated as distributed parameter control problems:**
 - Chemical plants**
 - Steel making plants**
 - Semiconductor manufacturing**
 - Environmental/Energy management control**
 - **Space Structures and Communication**
 - Large and multiple aperture telescopes**
 - Lightweight, shape maintaining antennas**
 - Large flexible structures**
 - Acoustic structural control**

Introduction & Motivation (Cont.)

- ***Microprocessor Control Element:*** Low internal noise and time stable; Complex control algorithms readily implemented; Control algorithms easily reconfigured; Self checking/built-in-test capable
- ***Distributed Parameter Systems:*** Accurate description of some systems which is required for satisfying stringent performance requirements
- ***H -Control Methodology:*** A useful framework for analyzing and synthesizing controllers which satisfy nominal and robust performance requirements
- ***Direct Sampled-Data Synthesis:*** Demonstrated benefits over indirect approaches; easier to trade desired performance against required sample rate
- ***Finite-Dimensional Controller:*** Required for engineering implementation

Introduction & Motivation (Cont.)

- ***Thesis Objective:***

- Design of near-optimal finite dimensional sampled-data controllers for distributed parameter systems**

- **Systematic design method w/guaranteed performance bounds**
 - **Applicable to a large class of DPS w/application**

- ***Typical Engineering Practice (Approximate/Design):***

- Method 1: Approximate PDE using FEM software;**

- Design controller using continuous-time methods**

- Discretize controller for sampled-data implementation**

- Method 2: Discretize FEM model and use discrete-time design methods**

- ***Difficulties with Current Practice:***

- **Heuristic model approximation/design**

- High sample rates**

- No performance guarantees**

Sampled Data Control of Distributed Parameter Systems Literature Search

- **One article found: "Adaptive Low-Gain Sampled-Data Control of DPS", *CDC*, December 1995.**
- **Related articles:**
 - **Sampled data control**
 - **DPS discrete-time control**
 - **Finite-dimensional controllers for infinite dimensional systems**
 - **others**

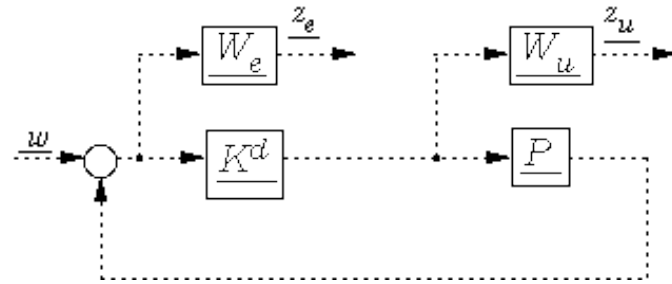
Contributions of Research

- **Systematic method for designing finite-dimensional sampled-data controllers for a large class of ~~SISO~~ MIMO distributed parameter systems**
- **Method based on finite-dimensional aproximants**
- **A priori computable approximant order**
- **Guaranteed performance bounds**
 -
 - **...More to come**
 -
 -

Sampled-Data Problem Formulation

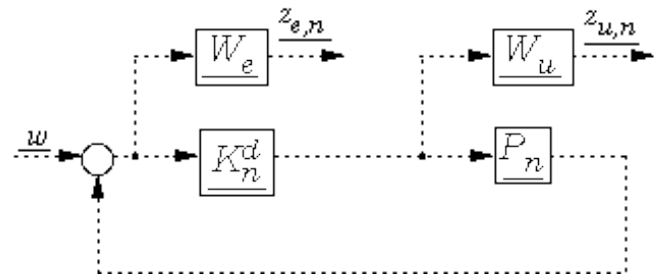
Optimal Performance

$$\begin{aligned} \mu_{opt} &= \inf_{K^d \text{ stabilizing}} \sup_{\|w\|_{\ell^2} = 1} \|z\|_{\ell^2} \\ &= \|\hat{t}\|_H \end{aligned}$$



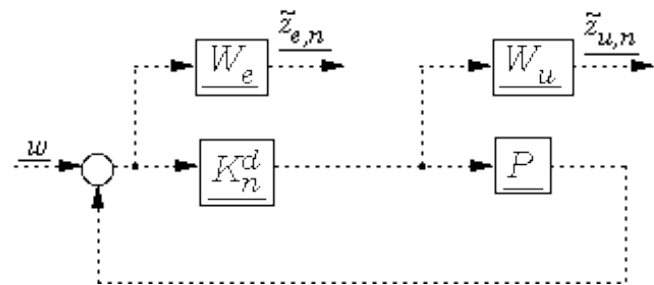
Expected Performance

$$\begin{aligned} \mu_n &= \inf_{K_n^d \text{ stabilizing}} \sup_{\|w\|_{\ell^2} = 1} \|z_n\|_{\ell^2} \\ &= \|\hat{t}_n\|_H \end{aligned}$$



Actual Performance

$$\begin{aligned} \tilde{\mu}_n &= \sup_{\|w\|_{\ell^2} = 1} \|\tilde{z}_n\|_{\ell^2} \\ &= \|\tilde{\hat{t}}_n\|_H \end{aligned}$$



Problems Addressed

1. Approximate/Design

Find conditions on performance measure and system approximants such that

$$\lim_n \tilde{\mu}_n = \mu_{opt}$$

2. Purely Finite Dimensional

Find conditions on performance measure and system approximants such that

$$\lim_n \mu_n = \mu_{opt}$$

Main Technical Ideas

- ***H*** -Optimal Sample-Data Control of Finite-Dimensional Systems
- ***Lifting of Continuous-Time System***
- ***H*** Finite-Dimensional Controller Synthesis for Continuous-Time Distributed Parameter Systems
- ***Well Behaved Performance Measures***
- ***Approximation Methods***

Results from H Optimal Control of Sampled-Data Systems (Finite Dimensional Plants)

- ***Objective:***

Direct controller synthesis for sampled-data systems which satisfy a certain closed loop H norm criterion with signal intersample behavior included

- ***Applicability:***

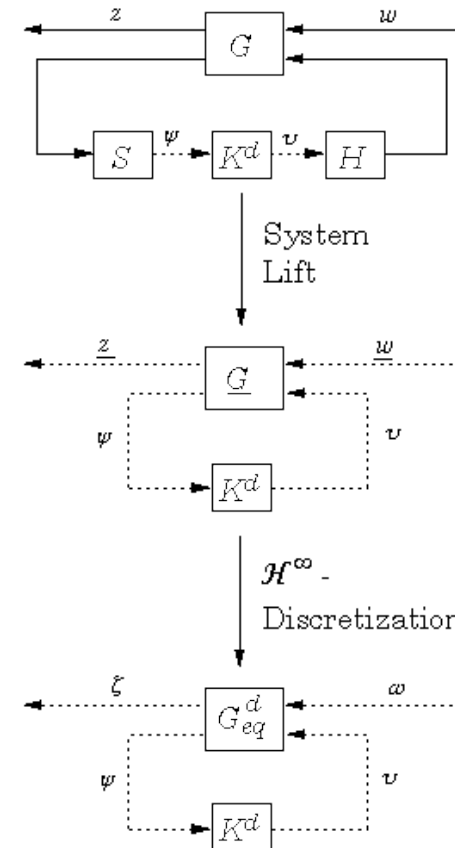
Finite dimensional linear time-invariant systems connected to controller through synchronized sampling and hold devices

- ***Key Concepts***

- **Continuous-time lift operator**
- **Operator-valued transfer functions**
- **H - Discretization**

H Optimal Control of Sampled-Data System Framework

- Finite dimensional plant connected to controller via sampling and hold devices results in periodic time-varying system
- Lifting system results in a discrete time-invariant system with infinite dimensional input and output spaces and with same induced norm as original system
- *H* - Discretization produces an induced norm equivalent discrete-time system with finite dimensional input and output



Transformation from Periodically Time-Varying to Discrete-Time Invariance via System Lift

- *Periodically time-varying system*

$$\begin{aligned} \dot{x}(k) &= a(k)x(k) + b(k)u(k), \quad x(0) = 0 \\ y(k) &= c(k)x(k) \end{aligned}$$

where

$$a(k) = \begin{cases} 0, & k \text{ even} \\ 1, & k \text{ odd} \end{cases}$$

0	0	0	0	0	0	...
1	0	0	0	0	0	...
1	1	0	0	0	0	...
0	0	1	0	0	0	...
0	0	1	1	0	0	...
0	0	0	0	1	0	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮

$$\begin{aligned} x_{(0)} &= \begin{pmatrix} x(0) \\ x(1) \end{pmatrix}, & x_{(1)} &= \begin{pmatrix} x(2) \\ x(3) \end{pmatrix}, \dots \\ x_{(0)} &= \begin{pmatrix} x(0) \\ x(1) \end{pmatrix}, & x_{(1)} &= \begin{pmatrix} x(2) \\ x(3) \end{pmatrix}, \dots \end{aligned}$$

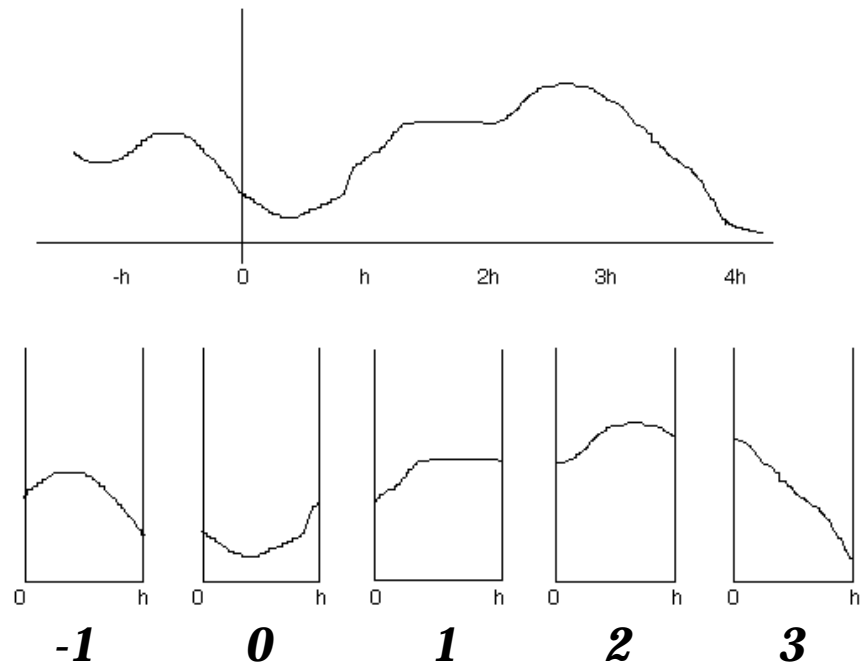
Lift Operator

- Lift operator is an isomorphism between Hilbert spaces, i.e.

$$L^2(\mathbb{R}) \cong L^2(\mathbb{Z}, K)$$

$$K = L^2([0, h), \mathbb{R}^n) \cong L^2[0, h)$$
- Maps continuous-time signals into discrete-time signals
- Preserves all standard algebraic and feedback interconnection operations
- Feedback stability preserved

Action of Lift Operator



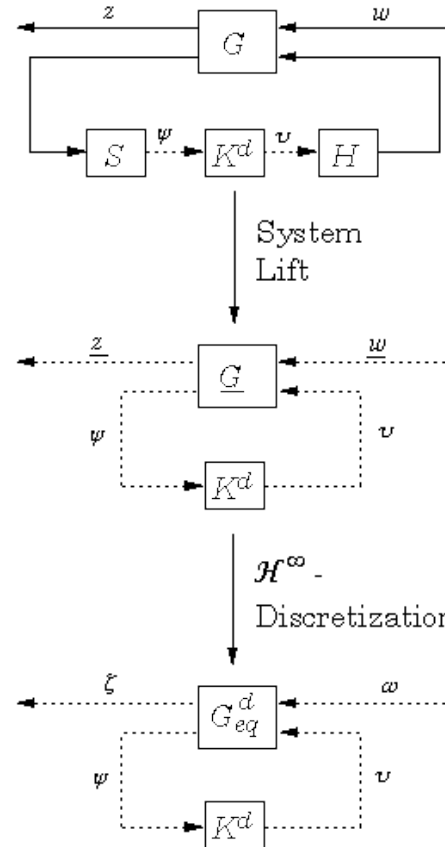
Induced Norm Equivalence

...

$$\|T\| = \sup_{\|w\|_{\ell^2}=1} \|z\|_{\ell^2}$$

$$\begin{aligned} \|T\| &= \sup_{\|w\|_{\ell^2}=1} \|z\|_{\ell^2} \\ &= \|\hat{t}\|_H = \sup_0^2 \|\hat{t}(e^i)\| \\ &= \|T\| \end{aligned}$$

$$\begin{aligned} \|T_{eq,d}\| &= \sup_{\|w\|_{\ell^2}=1} \|z\|_{\ell^2} \\ &= \|\hat{t}_{eq,d}\|_H \\ &= \|T\| \end{aligned}$$



H ***Optimal Control of Sampled-Data Systems*** ***Summary***

- **Applicable for finite dimensional plants**
- **Direct synthesis of digital controllers for sampled-data applications**
- **Methodology based on:**
 - **Norm preserving sampled-data system lift**
 - ***H* -norm preserving system discretization**

Continuous-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems

- ***Objective:***
Continuous-time near-optimal finite-dimensional controller synthesis methodology for continuous-time distributed parameter systems.
- ***Applicability:***
 - Stable MIMO distributed parameter systems
 - Continuous performance measure
- ***Key Ideas:***
 - Mixed sensitivity weighting function
 - Uniform plant approximants, P_n
 - Upper- and lower-semicontinuity at the plant, P
 - Control effort penalized in a nonsingular fashion

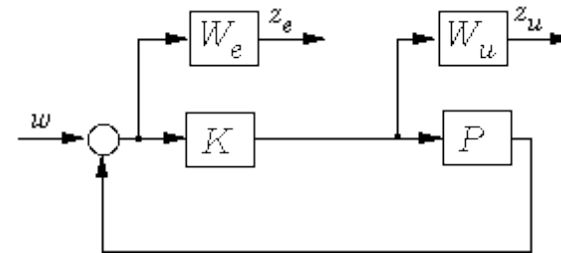
Continuous-Time Problem Formulation

...

Optimal Performance

$$\mu_{opt} = \inf_{K \text{ stabilizing}} \sup_{\|w\|_{L^2}=1} \|z\|_{L^2}$$

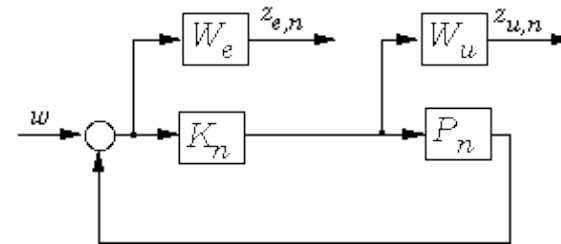
$$= \|\hat{t}\|_H$$



Expected Performance

$$\mu_n = \inf_{K_n \text{ stabilizing}} \sup_{\|w\|_{L^2}=1} \|z_n\|_{L^2}$$

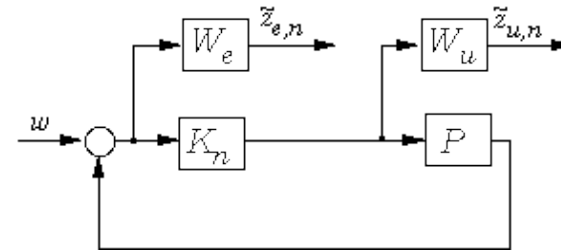
$$= \|\hat{t}_n\|_H$$



Actual Performance

$$\tilde{\mu}_n = \sup_{\|w\|_{L^2}=1} \|\tilde{z}_n\|_{L^2}$$

$$= \|\tilde{\hat{t}}_n\|_H$$



Problems Addressed

1. Approximate/Design

Find conditions on performance measure and system approximants such that

$$\lim_n \tilde{\mu}_n = \mu_{opt}$$

2. Purely Finite Dimensional

Find conditions on performance measure and system approximants such that

$$\lim_n \mu_n = \mu_{opt}$$

Continuous-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems Results

Given the *desired performance tolerance* $\delta > 0$, with

$$\frac{\delta}{\|W_e\| + 3 + \delta}$$

then

$$\mu_{opt} - \tilde{\mu}_n \leq \delta, \quad n \geq N \stackrel{def}{=} N(\delta, W_e, W_u)$$

and

- ***Theorem 1 (Solution to Purely Finite-Dimensional Problem)***

$$|\mu_n - \mu_{opt}| \leq 2\delta, \quad n \geq N \stackrel{def}{=} N(\delta, W_e, W_u)$$

Moreover,

$$\lim_n \mu_n = \mu_{opt}$$

Continuous-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems Results (Cont.)

- **Proposition (Stability of Actual Closed Loop Continuous-Time System: (P, K_n)).**

(P, K_n) results in a stable closed loop system $n \geq N = N(\epsilon, W_e, W_u)$

- **Theorem 2 (Solution to H_∞ Purely Approximate/Design Mixed-Sensitivity Problem).**

$$\mu_{opt} \leq \tilde{\mu}_n \leq \frac{\mu_{opt} + 3}{1 - \epsilon}, \quad n \geq N = N(\epsilon, W_e, W_u).$$

Moreover,

$$\lim_{n \rightarrow \infty} \tilde{\mu}_n = \mu_{opt}$$

Theorem 2 (Approximate/Design) Proof Outline

- **Mixed Sensitivity Performance Measure**

$$\tilde{\mu}_n = \left\| \begin{array}{c} W_e \\ W_u K(P_n, Q_n) (I - PK(P_n, Q_n))^{-1} \end{array} \right\|_H = \left\| \begin{array}{c} W_e (I - P_n Q_n) \\ W_u Q_n (I - (P_n - P) Q_n)^{-1} \end{array} \right\|_H$$

$n \quad N \quad N(, W_e, W_u)$

- **Left Inequality**

$$\tilde{\mu}_n \left\| \begin{array}{c} W_e (I - P_n Q_n) \\ W_u Q_n \end{array} \right\|_H \frac{1}{1 - \|(P_n - P) Q_n\|_H}$$

- **It's known that**

$$\mu_{opt} \leq \tilde{\mu}_n$$

Theorem 2 (Approximate/Design) Proof Outline (Cont.)

- **Choose**

$$Q_n \in RH$$

such that

$$\left\| \begin{array}{c} W_e(I - P_n Q_n) \\ W_u Q_n \end{array} \right\|_H \leq \mu_n + \epsilon$$

- **Apply Theorem 1 to obtain**

$$\left\| \begin{array}{c} W_e(I - P_n Q_n) \\ W_u Q_n \end{array} \right\|_H \leq \mu_n + \epsilon \leq \mu_{opt} + 3\epsilon$$

Theorem 2 (Approximate/Design) Proof Outline (Cont.)

- **Which yields**

$$\mu_{opt} \quad \tilde{\mu}_n = \frac{\mu_{opt} + 3}{1 - \|(P_n - P)Q_n\|_H}$$

- **Q_n is uniformly bounded**

$$\begin{aligned} \|Q_n\|_H &= \|W_u^{-1}W_u Q_n\|_H = \|W_u^{-1}\|_H \|W_u Q_n\|_H \\ &= \|W_u^{-1}\|_H (\mu_n + 3) = \|W_u^{-1}\|_H (\|W_e\|_H + 3) \\ &= B \end{aligned}$$

Theorem 2 (Approximate/Design) Proof Outline (Cont.)

- **So that**

$$\mu_{opt} \approx \tilde{\mu}_n = \frac{\mu_{opt} + 3}{1 - B \| (P_n - P) \|_H}$$

- **By construction of P_n**

$$\|P_n - P\|_H \leq \min \left\{ \frac{1}{\|W_e\|_H}, \frac{1}{B}, \frac{1}{B} \right\}, \quad n \leq N \leq N(d, W_e, W_u)$$

where

$$\|W_e\|_H = \frac{d}{d + 3 + d}$$

- **The proof then follows.**

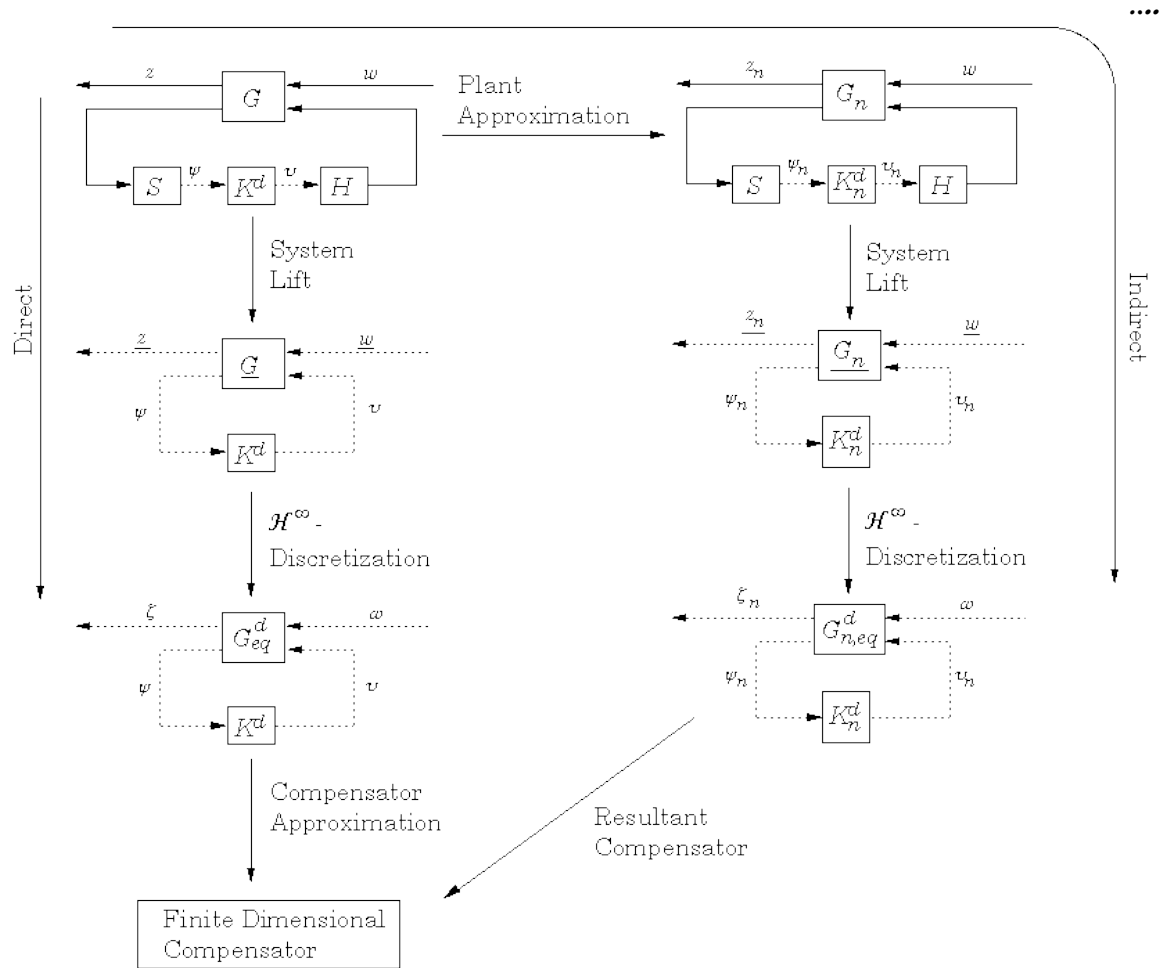
Continuous-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems Summary

- ***Systematic design of near-optimal finite-dimensional controllers for stable distributed parameter systems***
- ***Plant and controller operate in continuous-time***
- ***A priori computable approximant order based on desired performance tolerance***
- ***Finite dimensional techniques which yields accurate estimate of infinite dimensional system results***

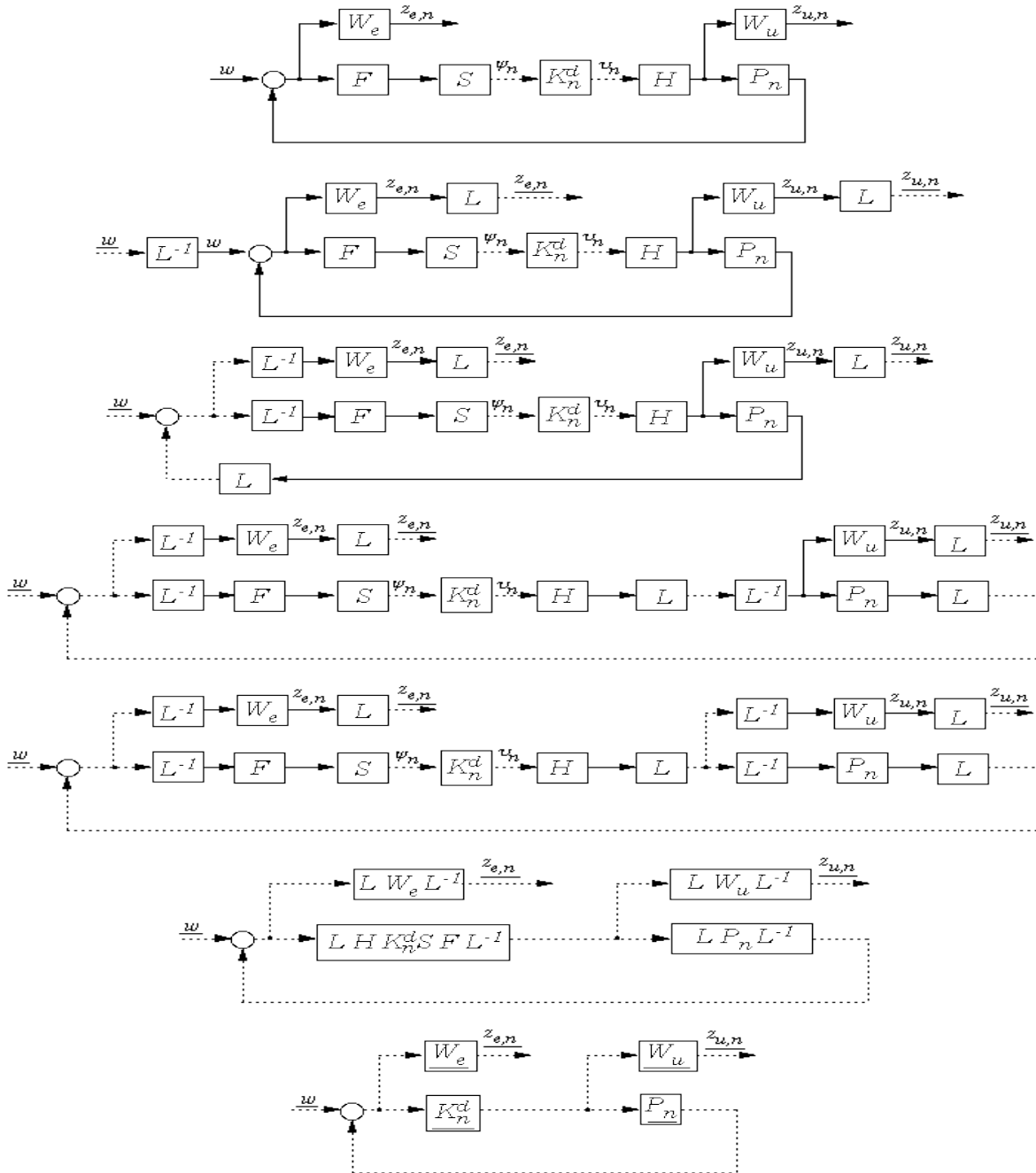
Discrete-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems

- ***Controller Synthesis Problem:***
Synthesize a finite-dimensional discrete-time controller for a continuous-time stable SISO distributed parameter plant such that the resultant closed-loop performance measure is within some desired tolerance of the optimal performance measure with intersample behavior included.
- ***Applied Methods***
 - Optimal control of sampled-data systems
 - Near-optimal finite-dimensional compensator synthesis for stable MIMO distributed parameter systems
- ***Key Assumptions:***
 - Same as in stated methods

Alternative Approaches to Solution



Reduction of Sampled-Data Problem



Sampled-Data Mixed-Sensitivity Performance Measures

- **Expected Performance**

$$\mu_n = \inf_{K_n^d \text{ stabilizing}} \|T_n\| = \inf_{K_n^d \text{ stabilizing}} \|\underline{T}_n\| = \inf_{K_n^d \text{ stabilizing}} \left\| \frac{W_e}{\underline{W}_u \underline{K}_n} \left(I - \underline{P}_n \underline{K}_n^d \right)^{-1} \right\|_H$$

- **Optimal Performance**

$$\mu_{opt} = \inf_{K^d \text{ stabilizing}} \|T\| = \inf_{K^d \text{ stabilizing}} \|\underline{T}\| = \inf_{K^d \text{ stabilizing}} \left\| \frac{W_e}{\underline{W}_u \underline{K}^d} \left(I - \underline{P} \underline{K}^d \right)^{-1} \right\|_H$$

- **Actual Performance**

$$\tilde{\mu}_n = \|\tilde{T}_n\| = \|\tilde{\underline{T}}_n\| = \left\| \frac{W_e}{\underline{W}_u \underline{K}_n^d} \left(I - \underline{P} \underline{K}_n^d \right)^{-1} \right\|_H$$

Summary and Future Directions

- **A systematic approach to synthesizing near-optimal discrete-time finite-dimensional controllers for continuous-time SISO MIMO stable distributed parameter plants which can be uniformly approximated by H functions**
- **Future work to include:**
 - **Extension to MIMO unstable systems**
 - **Examination of approximation methods**
 - **Application to diffusion process**
 - **Application to flexible beam**