Oral Comprehensive Examination and Ph.D. Dissertation Proposal Defense

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Outline

- *Introduction and Motivation*
- *Literature Survey*
- *Problem Formulation*
- *Solution Approach*
- *Summary & Future Directions*

Introduction & Motivation

- ***Proposed Thesis Topic:***

  H^∞-Optimal Control of Distributed Parameter Sampled-Data Systems

- ***Targeted Application Areas:***
  - **Industrial Control Processes many of which are formulated as distributed parameter control problems:**

    Chemical plants
    Steel making plants
    Semiconductor manufacturing
    Environmental/Energy management control

  - **Space Structures and Communication**

    Large and multiple aperture telescopes
    Lightweight, shape maintaining antennas
    Large flexible structures
    Acoustic structural control

Introduction & Motivation (Cont.)

- ***Microprocessor Control Element:*** **Low internal noise and time stable; Complex control algorithms readily implemented; Control algorithms easily reconfigured; Self checking/built-in-test capable**
- ***Distributed Parameter Systems:*** **Accurate description of some systems which is required for satisfying stringent performance requirements**
- ***H^∞-Control Methodology:*** **A useful framework for analyzing and synthesizing controllers which satisfy nominal and robust performance requirements**
- ***Direct Sampled-Data Synthesis:*** **Demonstrated benefits over indirect approaches; easier to trade desired performance against required sample rate**
- ***Finite-Dimensional Controller:*** **Required for engineering implementation**

Introduction & Motivation (Cont.)

- ***Thesis Objective:***

  Design of near-optimal finite dimensional sampled-data controllers for distributed parameter systems

  − Systematic design method w/guaranteed performance bounds

  − Applicable to a large class of DPS w/application

- ***Typical Engineering Practice (Approximate/Design):***

  Method 1: Approximate PDE using FEM software;
  Design controller using continuous-time methods
  Discretize controller for sampled-data implementation

  Method 2: Discretize FEM model and use discrete-time design methods

- ***Difficulties with Current Practice:***

  − Heuristic model approximation/design
  High sample rates
  No performance guarantees

Sampled Data Control of Distributed Parameter Systems Literature Search

- **One article found: "Adaptive Low-Gain Sampled-Data Control of DPS", *CDC*, December 1995.**
- **Related articles:**
  - **Sampled data control**
  - **DPS discrete-time control**
  - **Finite-dimensional controllers for infinite dimensional systems**
  - **others**

Contributions of Research

- **Systematic method for designing finite-dimensional sampled-data controllers for a large class of ~~SISO~~ MIMO distributed parameter systems**
- **Method based on finite-dimensional aproximants**
- **A priori computable approximant order**
- **Guaranteed performance bounds**

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- **...More to come**

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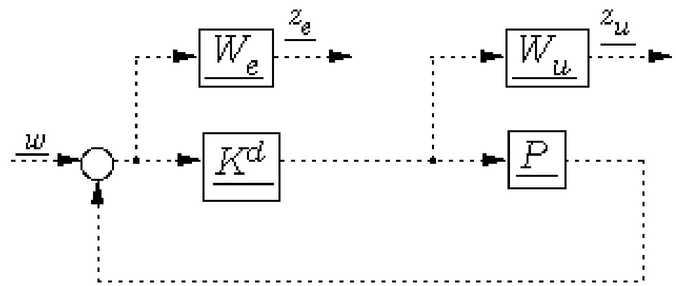
Sampled-Data Problem Formulation

Optimal Performance

$$\mu_{opt} \equiv \inf_{\underline{K}^d \, stabilizing} \left\{ \sup_{\|\underline{w}\|_{\ell^2}=1} \|\underline{z}\|_{\ell^2} \right\}$$

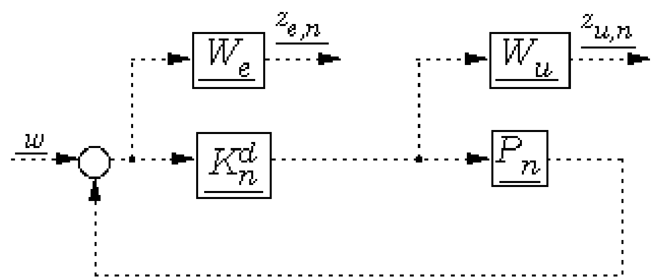
$$= \left\|\hat{\underline{t}}\right\|_{\boldsymbol{H}^\infty}$$



Expected Performance

$$\mu_n \equiv \inf_{\underline{K}_n^d \, stabilizing} \left\{ \sup_{\|\underline{w}\|_{\ell^2}=1} \left\|\underline{z}_n\right\|_{\ell^2} \right\}$$

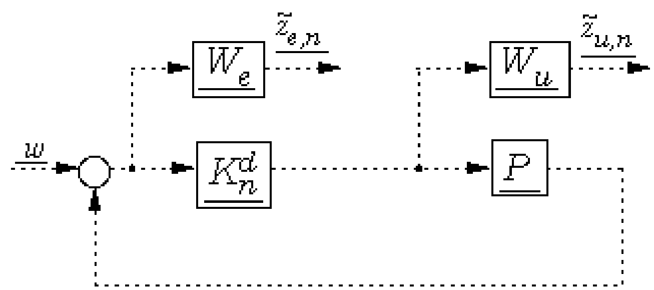
$$= \left\|\hat{\underline{t}}_n\right\|_{\boldsymbol{H}^\infty}$$



Actual Performance

$$\tilde{\mu}_n \equiv \sup_{\|\underline{w}\|_{\ell^2}=1} \left\|\underline{\tilde{z}}_n\right\|_{\ell^2}$$

$$= \left\|\hat{\underline{\tilde{t}}}_n\right\|_{\boldsymbol{H}^\infty}$$



Problems Addressed

1. ***Approximate/Design***

   Find conditions on performance measure and system approximants such that

$$\lim_{n \to \infty} \tilde{\mu}_n = \mu_{opt}$$

2. ***Purely Finite Dimensional***

   Find conditions on performance measure and system approximants such that

$$\lim_{n \to \infty} \mu_n = \mu_{opt}$$

Main Technical Ideas

- *H^{∞}-Optimal Sample-Data Control of Finite-Dimensional Systems*
- *Lifting of Continuous-Time System*
- *H^{∞} Finite-Dimensional Controller Synthesis for Continuous-Time Distributed Parameter Systems*
- *Well Behaved Performance Measures*
- *Approximation Methods*

Results from H^{∞} Optimal Control of Sampled-Data Systems (Finite Dimensional Plants)

- ***Objective:***

  Direct controller synthesis for sampled-data systems which satisfy a certain closed loop H^{∞} norm criterion with signal intersample behavior included

- ***Applicability:***

  Finite dimensional linear time-invariant systems connected to controller through synchronized sampling and hold devices

- ***Key Concepts***

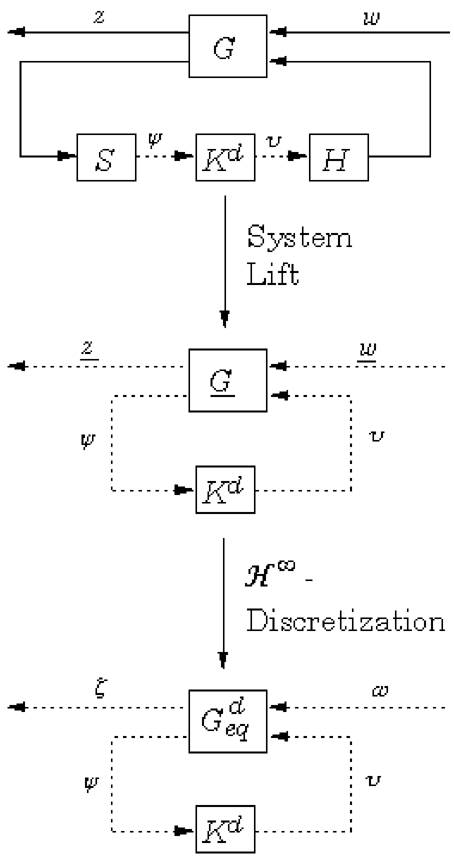
  - **Continuous-time lift operator**
  - **Operator-valued transfer functions**
  - **H^{∞}- Discretization**

H^{∞} *Optimal Control of Sampled-Data System Framework*

- **Finite dimensional plant connected to controller via sampling and hold devices results in periodic time-varying system**

- **Lifting system results in a discrete time-invariant system with infinite dimensional input and output spaces and with same induced norm as original system**

- H^{∞} **- Discretization produces an induced norm equivalent discrete-time system with finite dimensional input and output**



Transformation from Periodically Time-Varying to Discrete-Time Invariance via System Lift

- ***Periodically time-varying system***

$$\dot{\xi}(k) = a(k)\xi(k) + \vartheta(k), \quad \xi(0) = 0$$

$$\psi(k) = \xi(k)$$

where

$$a(k) = \begin{cases} 0, & k \text{ even} \\ 1, & k \text{ odd} \end{cases}$$

$$\left[\begin{array}{cc|cc|cc|c} 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\ \hline 0 & 0 & 1 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}\right]$$

$$\underline{\upsilon}(0) = \begin{bmatrix} \upsilon(0) \\ \upsilon(1) \end{bmatrix} \quad \underline{\upsilon}(1) = \begin{bmatrix} \upsilon(2) \\ \upsilon(3) \end{bmatrix}, \ldots$$

$$\underline{\psi}(0) = \begin{bmatrix} \psi(0) \\ \psi(1) \end{bmatrix} \quad \underline{\psi}(1) = \begin{bmatrix} \psi(2) \\ \psi(3) \end{bmatrix}, \ldots$$

Lift Operator

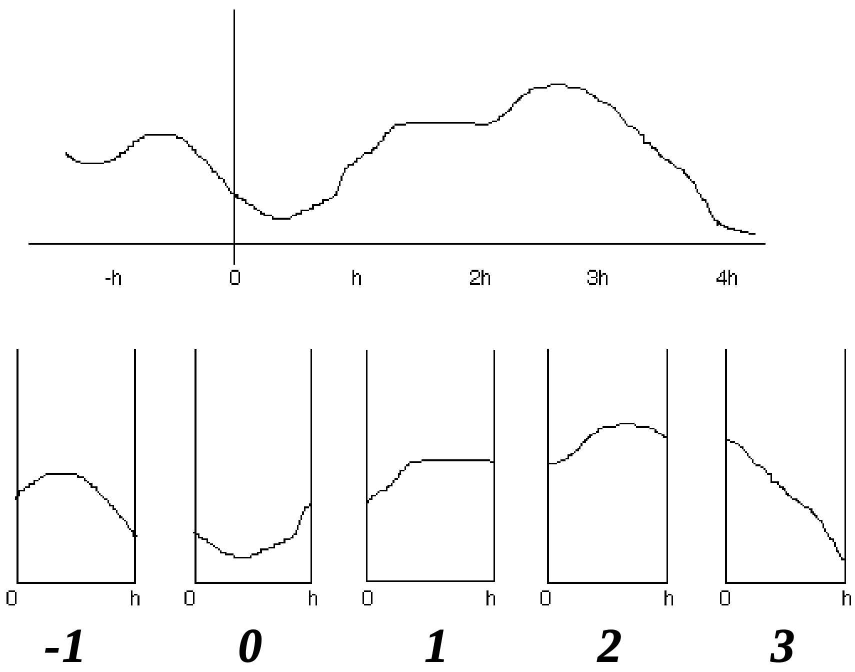
- **Lift operator is an isomorphism between Hilbert spaces, i.e.**

  $$\boldsymbol{L}^2(\quad) \equiv \boldsymbol{L}^2\left(\quad, \quad^{n}\right) \to \ell^2(\boldsymbol{Z},\boldsymbol{K})$$

  $$\boldsymbol{K} \equiv \boldsymbol{L}^2\left([0,h), \quad^{n}\right) \equiv \boldsymbol{L}^2\,[0,h)$$

- **Maps continuous-time signals into discrete-time signals**
- **Preserves all standard algebraic and feedback interconnection operations**
- **Feedback stability preserved**

Action of Lift Operator



Induced Norm Equivalence

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$$\|T\| = \sup_{\|w\|_{L^2}=1} \|z\|_{L^2}$$

$$\|\underline{T}\| = \sup_{\|\underline{w}\|_{\ell^2}=1} \|\underline{z}\|_{\ell^2}$$

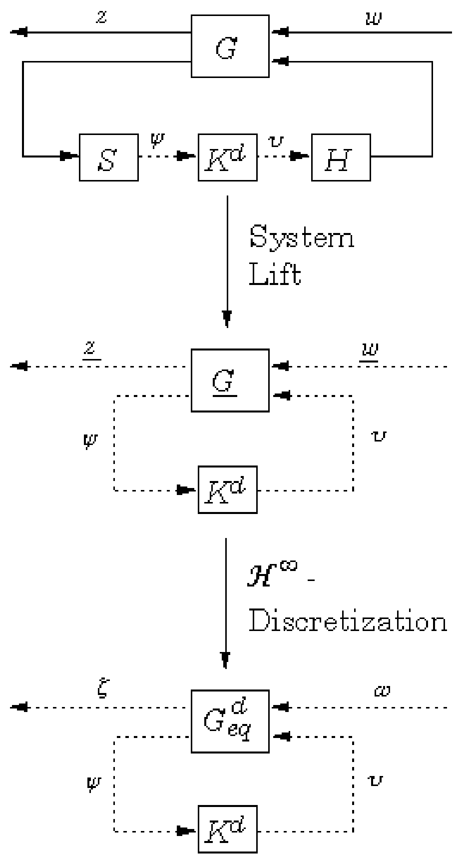
$$= \|\hat{\underline{t}}\|_{H^\infty} = \sup_{0\le\theta\le 2\pi} \|\hat{\underline{t}}(e^{j\theta})\|$$

$$= \|T\|$$

$$\|T_{eq,d}\| = \sup_{\|\omega\|_{\ell^2}=1} \|\zeta\|_{\ell^2}$$

$$= \|\hat{t}_{eq,d}\|_{H^\infty}$$

$$= \|T\|$$



H^{∞} *Optimal Control of Sampled-Data Systems Summary*

- **Applicable for finite dimensional plants**
- **Direct synthesis of digital controllers for sampled-data applications**
- **Methodology based on:**
  - **Norm preserving sampled-data system lift**
  - H^{∞}**-norm preserving system discretization**

Continuous-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems

- ***Objective:***

  Continuous-time near-optimal finite-dimensional controller synthesis methodology for continuous-time distributed parameter systems.

- ***Applicability:***
  - **Stable MIMO distributed parameter systems**
  - **Continuous performance measure**

- ***Key Ideas:***
  - **Mixed sensitivity weighting function**
  - **Uniform plant approximants, P_n**
  - **Upper- and lower-semicontinuity at the plant, P**
  - **Control effort penalized in a nonsingular fashion**

Continuous-Time Problem Formulation

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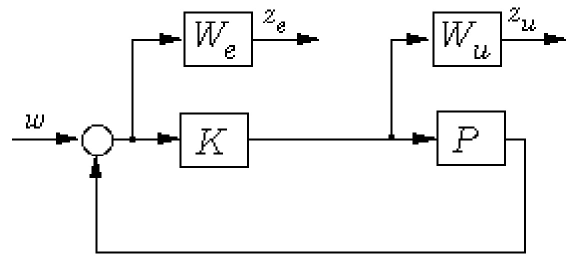
Optimal Performance

$$\mu_{opt} \equiv \inf_{K \text{ stabilizing}} \left\{ \sup_{\|w\|_{\mathbf{L}^2}=1} \|z\|_{\mathbf{L}^2} \right\}$$

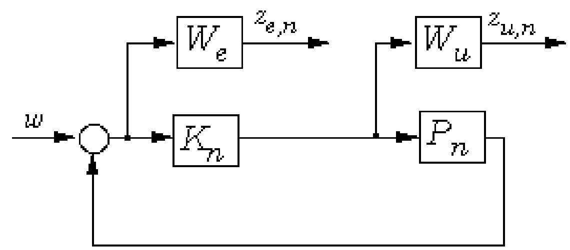
$$= \left\|\hat{t}\right\|_{\mathbf{H}^\infty}$$



Expected Performance

$$\mu_n \equiv \inf_{K_n \text{ stabilizing}} \left\{ \sup_{\|w\|_{\mathbf{L}^2}=1} \|z_n\|_{\mathbf{L}^2} \right\}$$

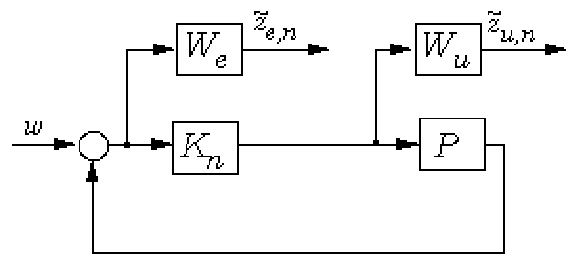
$$= \left\|\hat{t}_n\right\|_{\mathbf{H}^\infty}$$



Actual Performance

$$\tilde{\mu}_n \equiv \sup_{\|w\|_{\mathbf{L}^2}=1} \|\tilde{z}_n\|_{\mathbf{L}^2}$$

$$= \left\|\tilde{\hat{t}}_n\right\|_{\mathbf{H}^\infty}$$



Problems Addressed

1. ***Approximate/Design***

   Find conditions on performance measure and system approximants such that

$$\lim_{n \to \infty} \tilde{\mu}_n = \mu_{opt}$$

2. ***Purely Finite Dimensional***

   Find conditions on performance measure and system approximants such that

$$\lim_{n \to \infty} \mu_n = \mu_{opt}$$

Continuous-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems Results

Given the *desired performance tolerance* $\varepsilon_d > 0$**, with**

$$\varepsilon \leq \frac{\varepsilon_d}{\|W_e\| + 3 + \varepsilon_d}$$

then

$$\mu_{opt} \leq \tilde{\mu}_n \leq \mu_{opt} + \varepsilon_d, \quad n \geq N \stackrel{def}{=} N(\varepsilon, W_e, W_u)$$

and

- ***Theorem 1 (Solution to Purely Finite-Dimensional Problem)***

$$\left|\mu_n - \mu_{opt}\right| \leq 2\varepsilon, \quad n \geq N \stackrel{def}{=} N(\varepsilon, W_e, W_u).$$

Moreover,

$$\lim_{n \to \infty} \mu_n = \mu_{opt}$$

Continuous-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems Results (Cont.)

- ***Proposition (Stability of Actual Closed Loop Continuous-Time System:*** (P, K_n)***).***

  (P, K_n) **results in a stable closed loop system** $n \geq N \overset{def}{=} N(\varepsilon, W_e, W_u)$

- ***Theorem 2 (Solution to*** H^{∞} ***Purely Approximate/Design Mixed-Sensitivity Problem).***

$$\mu_{opt} \leq \tilde{\mu}_n \leq \frac{\mu_{opt} + 3\varepsilon}{1 - \varepsilon}, \quad n \geq N \overset{def}{=} N(\varepsilon, W_e, W_u).$$

Moreover,

$$\lim_{n \to \infty} \tilde{\mu}_n = \mu_{opt}$$

Theorem 2 (Approximate/Design) Proof Outline

- **Mixed Sensitivity Performance Measure**

$$\tilde{\mu}_n = \left\| \begin{pmatrix} W_e \\ W_u K(P_n, Q_n) \end{pmatrix} \left(I - PK(P_n, Q_n)\right)^{-1} \right\|_{H^\infty} = \left\| \begin{pmatrix} W_e(I - P_n Q_n) \\ W_u Q_n \end{pmatrix} \left(I - (P_n - P) Q_n\right)^{-1} \right\|_{H^\infty}$$

$$n \geq N \equiv N(\varepsilon, W_e, W_u)$$

- **Left Inequality**

$$\tilde{\mu}_n \leq \left\| \begin{pmatrix} W_e(I - P_n Q_n) \\ W_u Q_n \end{pmatrix} \right\|_{H^\infty} \frac{1}{1 - \left\| (P_n - P) Q_n \right\|_{H^\infty}}$$

- **It's known that**

$$\mu_{opt} \leq \tilde{\mu}_n$$

Theorem 2 (Approximate/Design) Proof Outline (Cont.)

- ***Choose***

$$Q_n \quad RH^{\infty}$$

such that

$$\left\| \begin{bmatrix} W_e\left(I - P_n Q_n\right) \\ W_u Q_n \end{bmatrix} \right\|_{H^{\infty}} \leq \mu_n + \varepsilon$$

- ***Apply Theorem 1 to obtain***

$$\left\| \begin{bmatrix} W_e\left(I - P_n Q_n\right) \\ W_u Q_n \end{bmatrix} \right\|_{H^{\infty}} \leq \mu_n + \varepsilon \leq \mu_{opt} + 3\varepsilon$$

Theorem 2 (Approximate/Design) Proof Outline (Cont.)

- ***Which yields***

$$\mu_{opt} \le \tilde{\mu}_n \le \frac{\mu_{opt} + 3\varepsilon}{1 - \left\| (P_n - P) Q_n \right\|_{\boldsymbol{H}^\infty}}$$

- ***Q_n is uniformly bounded***

$$\begin{aligned} \|Q_n\|_{\boldsymbol{H}^\infty} &= \left\| W_u^{-1} W_u Q_n \right\|_{\boldsymbol{H}^\infty} \le \left\| W_u^{-1} \right\|_{\boldsymbol{H}^\infty} \left\| W_u Q_n \right\|_{\boldsymbol{H}^\infty} \\ &\le \left\| W_u^{-1} \right\|_{\boldsymbol{H}^\infty} (\mu_n + \varepsilon) \le \left\| W_u^{-1} \right\|_{\boldsymbol{H}^\infty} \left(\|W_e\|_{\boldsymbol{H}^\infty} + \varepsilon \right) \\ &= B \end{aligned}$$

Theorem 2 (Approximate/Design) Proof Outline (Cont.)

- ***So that***

$$\mu_{opt} \le \tilde{\mu}_n \le \frac{\mu_{opt} + 3\varepsilon}{1 - B\left\|(P_n - P)\right\|_{\mathcal{H}^\infty}}$$

- ***By construction of P_n***

$$\left\|P_n - P\right\|_{\mathcal{H}^\infty} \le \min\left\{\frac{\varepsilon}{\left\|W_e\right\|_{\mathcal{H}^\infty} B}, \frac{\varepsilon}{B}\right\}, \quad n \ge N \equiv N(\varepsilon, W_e, W_u)$$

where

$$\varepsilon \le \frac{\varepsilon_d}{\left\|W_e\right\|_{\mathcal{H}^\infty} + 3 + \varepsilon_d}$$

- ***The proof then follows.***

Continuous-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems Summary

- *Systematic design of near-optimal finite-dimensional controllers for stable distributed parameter systems*
- *Plant and controller operate in continuous-time*
- *A priori computable approximant order based on desired performance tolerance*
- *Finite dimensional techniques which yields accurate estimate of infinite dimensional system results*

Discrete-Time Finite-Dimensional Controller Synthesis for Distributed Parameter Systems

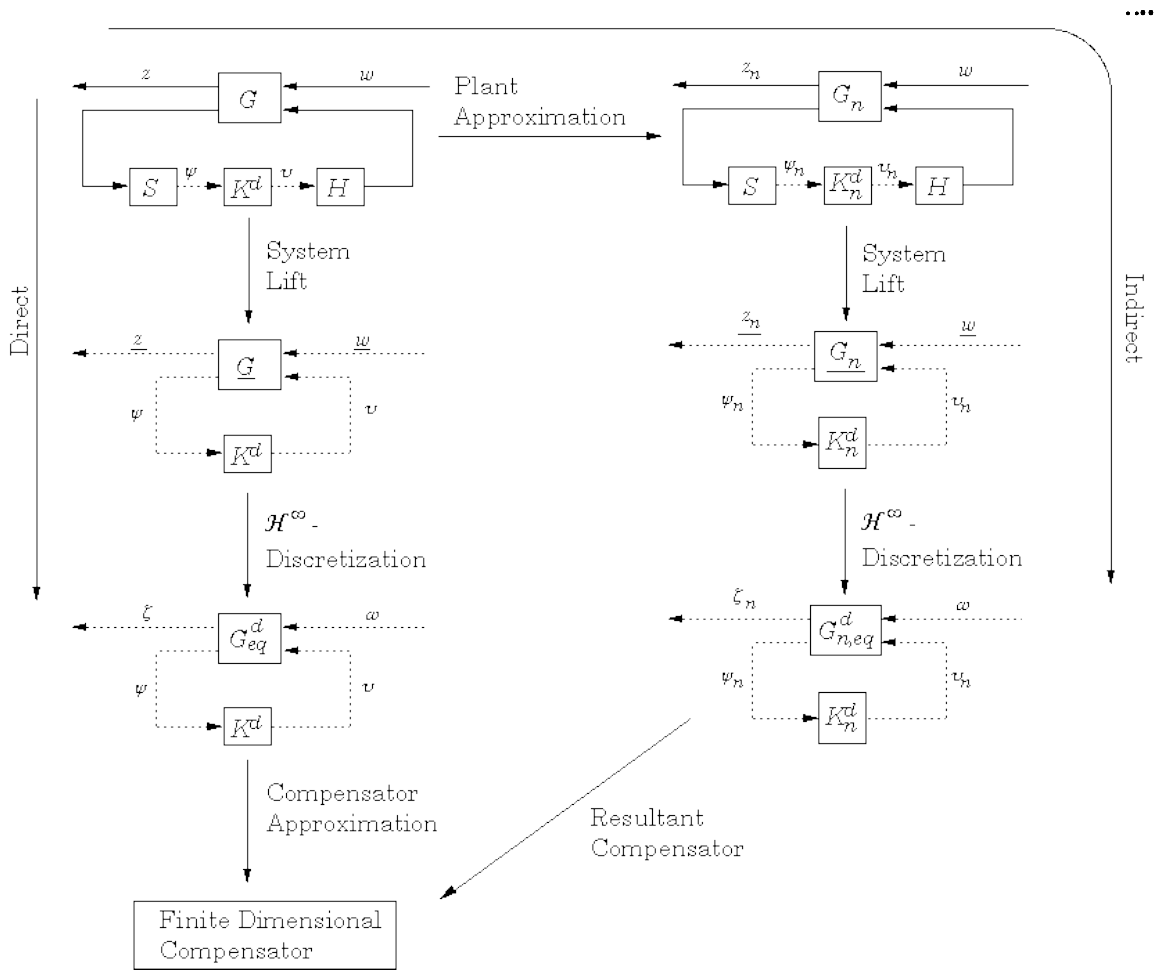
- ***Controller Synthesis Problem:***

  Synthesize a finite-dimensional discrete-time controller for a continuous-time stable SISO distributed parameter plant such that the resultant closed-loop performance measure is within some desired tolerance of the optimal performance measure with intersample behavior included.

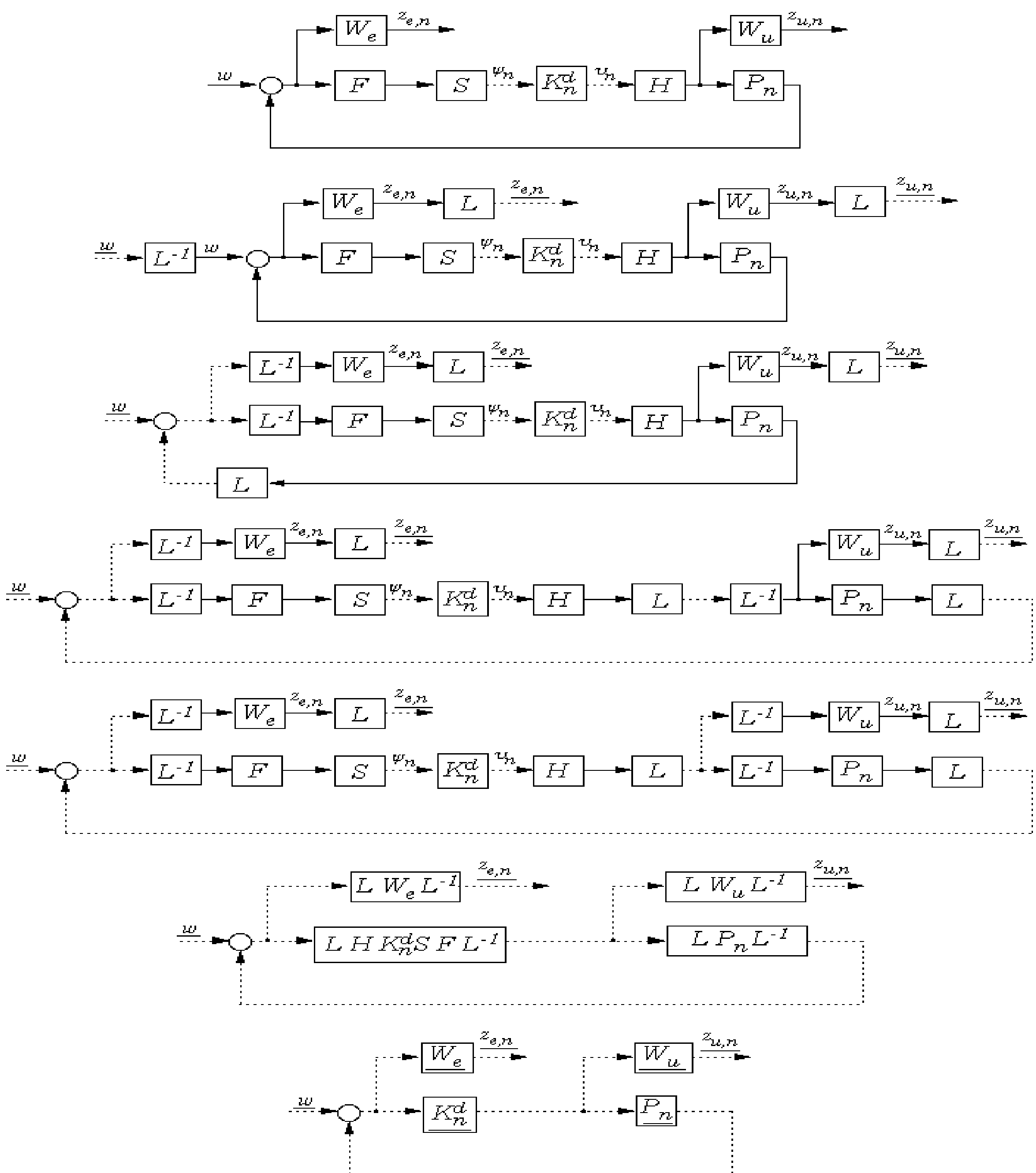
- ***Applied Methods***
  - **Optimal control of sampled-data systems**
  - **Near-optimal finite-dimensional compensator synthesis for stable MIMO distributed parameter systems**

- ***Key Assumptions:***
  - **Same as in stated methods**

Alternative Approaches to Solution



Reduction of Sampled-Data Problem



Sampled-Data Mixed-Sensitivity Performance Measures

- **Expected Performance**

$$\mu_n = \inf_{K_n^d \, stabilizing} \left\| T_n \right\| = \inf_{K_n^d \, stabilizing} \left\| \underline{T}_n \right\| = \inf_{K_n^d \, stabilizing} \left\| \begin{pmatrix} \underline{W_e} \\ \underline{W_u} \, \underline{K}_n^d \end{pmatrix} \left(I - \underline{P}_n \underline{K}_n^d \right)^{-1} \right\|_{H^\infty}$$

- **Optimal Performance**

$$\mu_{opt} = \inf_{K^d \, stabilizing} \left\| T \right\| = \inf_{K^d \, stabilizing} \left\| \underline{T} \right\| = \inf_{K^d \, stabilizing} \left\| \begin{pmatrix} \underline{W_e} \\ \underline{W_u} \, \underline{K}^d \end{pmatrix} \left(I - \underline{P} \, \underline{K}^d \right)^{-1} \right\|_{H^\infty}$$

- **Actual Performance**

$$\tilde{\mu}_n = \left\| \tilde{T}_n \right\| = \left\| \underline{\tilde{T}}_n \right\| = \left\| \begin{pmatrix} \underline{W_e} \\ \underline{W_u} \, \underline{K}_n^d \end{pmatrix} \left(I - \underline{P} \, \underline{K}_n^d \right)^{-1} \right\|_{H^\infty}$$

Summary and Future Directions

- **A systematic approach to synthesizing near-optimal discrete-time finite-dimensional controllers for continuous-time ~~SISO~~ MIMO stable distributed parameter plants which can be uniformly approximated by H^{∞} functions**

- **Future work to include:**
  - **Extension to MIMO unstable systems**
  - **Examination of approximation methods**
  - **Application to diffusion process**
  - **Application to flexible beam**